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DYNAMICS OF BANDITRY ACTIVITIES: A MATHEMATICAL MODELING APPROACH

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Abstract Banditry represents a significant worldwide social issue; in light of this, a mathematical model for the dynamics of banditry activities was developed. The model is intended to control banditry activities in society. The model encompasses six population classes and incorporates preventive measures such as preventing susceptible individuals from being kidnapped and taking the captured bandits into jail or detention. Bandits-free and bandits persistence were established. Also, the threshold parameter (Reproduction Number (R_c)) used to measure the level of banditry activities was established, as ($R_c < 1$), banditry activities die out from the dynamical system, and if $R_c > 1$ the banditry persists $R_c > 1$, in the system. Numerical results demonstrate that banditry is effectively curbed when $R_c < 1$ and persists otherwise. Stability analyses reveal the local asymptotic stability of the bandit-free equilibrium, further supported by global stability analysis employing the Lyapunov theorem. Empirical findings suggest that incarcerating captured bandits significantly outperforms lethal measures or preventing susceptible individuals from being kidnapped.

MSC: 93A30; 97M10
Keywords: Modeling, Banditry, Reproduction Number, Local and Global stability

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1. Introduction

Banditry is an act of organized crime committee by outlaws, typically involving the threats or use of violence [1]. The roots of banditry often lie in complex social, economic, and political factors, varying from one context to another. Understanding the dynamics of banditry and developing effective strategies to combat it require an interdisciplinary approach that considers a multitude of variables, including access to education and employment opportunities, geographical landscapes, and the presence of law enforcement, justice systems, and more importantly Government effort to eradicate banditry. The inherent complexity of this phenomenon necessitates a more systematic and data-driven approach, where mathematical modeling plays an important role. Prior attempts to combat banditry have mostly depended on personal stories, historical narratives, and qualitative assessments. Although these methods have yielded important discoveries, their capacity to convey the dynamic and ever-changing character of the phenomena is restricted. Conventional approaches have frequently failed to identify new clusters before they appear, to explain what leads to violent episodes, or to assess how well treatments work.

Bandits target rural residents, often ambushing drivers and killing those resisting kidnapping or refusing ransom. Banditry is a violent crime involving armed robberies and threats. In northern Nigerian states, over 21 million people live in areas with banditry crimes. The discovery of gold mines and illegal miners' activities have worsened armed bandits' presence in the northwest [2] endeavor to contribute to the growing body of research that utilizes mathematical models to address complex societal issues, specifically, to illuminate the inner workings of banditry. Within this rich and dynamic context, this research aims to unravel the mysteries of banditry and offer insights that can inform more effective strategies for prevention and control. Using a quantitative, data-driven methodology, this study seeks to go beyond the confines of traditional analysis, providing new resources to help scholars, law enforcement, and politicians tackle the complex problems that banditry presents. By delving into mathematical modeling, it aims to offer a more lucid perspective on the banditry environment, enabling us to make more informed and focused actions [13] developed a mathematical model of illicit drugs with banditry compartments. The model population is divided into six compartments. Also, [7] present a mathematical model for controlling the spread of armed banditry in Nigeria. The model consists of a system of five ordinary differential equations that take into account the dynamics of susceptible individuals $S(t)$ and $E(t)$, informants $I(t)$, bandits $B(t)$, and recovered individuals $R(t)$. The authors incorporate two time-dependent controls, namely job creation and efforts to make armed banditry unprofitable, into the model and analyze their effectiveness in decreasing the population profile of informants and bandits. The authors use the Pontryagin Maximum Principle (PMP) to characterize and simulate the optimal control model using the Forward-Backward Sweep Method of the fourth-order Runge-Kutta scheme. Also, [4] investigate the global stability of illicit drug use dynamic with banditry compartment using a dynamic system theory approach.

Among Several researchers who developed several mathematical models that aimed at curbing banditry and terrorism activities in Nigeria are: [2],[3],[11],[17], and [19]. Considering the high rate of Banditry activities, especially kidnapping, criminal attacks, and cattle rustling, the author is highly motivated and undertakes the research to develop a model that will curb the menace of Banditry activities. Thus, this work tends to develop a mathematical model of banditry incorporating detention of bandits and Government intervention.

2. Model Formulation

The model is sub-divided into the mutually exclusive sub-populations as follows: The susceptible individuals (S), the Bandits (B), the Captives (C), individuals undergoing Detention (D), the Freed individuals who regain freedom from captivity (F), and Government efforts to eradicate banditry (G).
The population of the susceptible (S), are generated from daily recruitment through birth or immigration at the rate Λ, they decrease due to kidnapping rate, movement to captives class (C), at the rate λ_1, and to the bandits class (B), at the rate of λ_2, and also by natural death μ. The population of the bandit class (B) is generated from susceptible λ_1 and diminishes by the death of bandits by the government at the rate of $\delta_3 G$, natural death at the rate μ, or progression into the detention class (D) at the rate α. The captives individuals (C) are generated through migration from the susceptible class (S) at the rate λ_2, and decreases by the death of captives by bandits at the rate δ_1 and collateral damage $\delta_2 G$, the class also reduces through progression to the freed individuals (F), at the rate τ_1 and $\tau_2 G$, the class also reduces due to natural death at the rate μ. The freed individuals (F) are those that regained freedom from captivity by either paying ransom at the rate τ_1, or government effort to rescue the captives at the rate τ_2. The class diminishes by natural death at the rate μ. The detention class (D) is generated by the progression of bandits at the rate α from the bandit class (B), and the class diminishes due to natural death at the rate μ.
Table 1 gives the meaning of the parameters and Figure 1 gives the schematic diagram of the model.

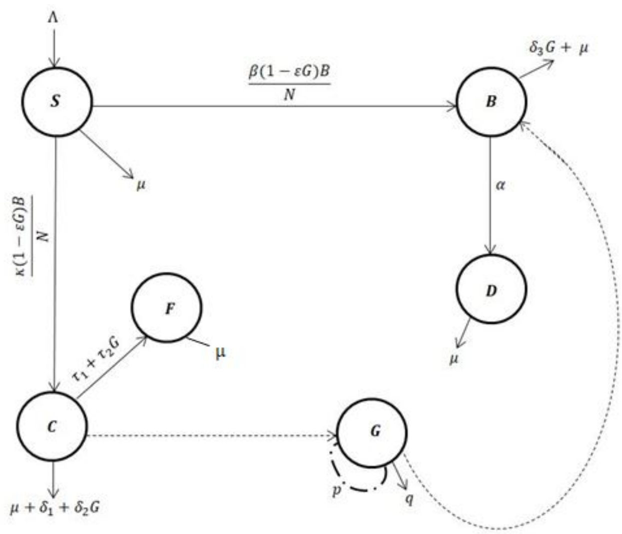

Figure 1. Schematic diagram for Banditry Model

The model equations are derived as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S - \frac{\kappa(1-\varepsilon G)BS}{N} - \frac{\beta(1-\varepsilon G)BS}{N},\\
\frac{dB}{dt} &= \frac{\beta(1-\varepsilon G)BS}{N} - (\delta_3 G + \mu + \alpha)\, B,\\
\frac{dC}{dt} &= \frac{\kappa(1-\varepsilon G)BS}{N} - (\tau_1 + \tau_2 G)C - (\mu + \delta_1 + \delta_2 G)C,\\
\frac{dF}{dt} &= (\tau_1 + \tau_2 G)C - \mu F,\\
\frac{dD}{dt} &= \alpha B - \mu D,\\
\frac{dG}{dt} &= p - qG.
\end{aligned} \tag{2.1}$$

Let

$$\lambda_1 = \frac{\beta(1-\varepsilon G)BS}{N}, \quad \lambda_2 = \frac{\kappa(1-\varepsilon G)BS}{N}.$$

Table 1. Description of Variables and Parameters

Variables/ Parameters	**Definition**
$S(t)$	Susceptible individuals
$B(t)$	Bandits
$C(t)$	Captives
$F(t)$	Freed individuals
$D(t)$	Individuals undergoing Detention
$G(t)$	Government Effort
Λ	recruitment rate into the susceptible population
κ	kidnapping rate
β	effective contact rate
ε_1	Government efforts in preventing Susceptible individuals from joining Bandit
ε_2	Government efforts in preventing Susceptible individuals from being kidnapped
α	rate of bandits moving to detention
τ_1	regain of freedom from captivity by either ransom or escape
τ_2	rescued by Government
δ_1	death of captives by bandits
δ_2	rate at which Government kills captives as collateral damage
δ_3	rate at which Government kills Bandits
μ	natural death rate

3. Basic Properties of the Model

3.1. Invariant region

The population size can be determined by the linear differential equation of the model formulated

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dB}{dt} + \frac{dC}{dt} + \frac{dF}{dt} + \frac{dD}{dt} + \frac{dG}{dt}, \tag{3.1}$$

i.e.

$$\frac{dN}{dt} = \Lambda - \mu N - (\delta_3 GB + \delta_1 C + \delta_2 GC),$$

such that

$$\frac{dN}{dt} \leq \Lambda - \mu N.$$

Since $N = S + B + C + F + D + G$ and the above equation resolved to linear differential equations of the form:

$$\frac{dN}{dt} + \mu N \leq \Lambda.$$

Theorem 3.1. *The solution of the system of the model in equation (2.1) is feasible for $t > 0$ if they are in the invariant region Ω.*

Proof. Let $(S, B, C, F, D, G) \in R^6$ be the solution of the system with non-negative initial conditions using an integrating factor

$$I.F. = e^{\int pdt} = e^{\mu t + C} = e^{\mu t} \cdot e^C = Ae^{\mu t},$$

$$Ae^{\mu t}\frac{dN}{dt} = \int Ae^{\mu t}\Lambda, \tag{3.2}$$

i.e.

$$\int Ae^{\mu t} dN = \int Ae^{\mu t}\Lambda dt, \tag{3.3}$$

i.e.

$$Ae^{\mu t}N = \frac{\Lambda Ae^{\mu t}}{\mu} + C, \tag{3.4}$$

such that

$$N(t) = \frac{\Lambda}{\mu} + Ce^{-\mu t}, \tag{3.5}$$

at $t = 0$ the initial population will become

$$N(0) = \frac{\Lambda}{\mu} + C, \tag{3.6}$$

where C is constant.

$$N(t) = \frac{\Lambda}{\mu}\left(1 - e^{-\mu t}\right) + N_0 e^{-\mu t}. \tag{3.7}$$

$$N(0) \leq \frac{\Lambda}{\mu}, \tag{3.8}$$

at $t \to \infty$ in (-) the human population N approaches $C = \dfrac{\Lambda}{\mu}$ i.e. $N \to C$, where $C = \dfrac{\Lambda}{\mu}$ is the caring capacity.

Hence all feasible solution sets of the population of the model system in (2.1) entered the region

$$\Omega = \{(S, B, C, F, D, G) \in R_+^6 : S + B + C + F + D + G \geq 0,\, N \leq \frac{\Lambda}{\mu}\}. \tag{3.9}$$

Hence it is positively invariant set under the flow induced by the model (2.1), hence the model is sociologically well-posed in the domain. Hence, the result satisfies the existence, continuity, and uniqueness of the system [16] as well as [18]. ∎

Theorem 3.2. *All the solutions of the systems (2.1) are positive for all time $t \geq 0$ provided that the initial conditions are positive. For the model systems (2.1), the region Ω is positively invariant and all solutions stating in Ω approach or stay in Ω.*

Proof. From the first equation in (2.1)

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - \mu S - \frac{\kappa(1-\varepsilon G)BS}{N} - \frac{\beta(1-\varepsilon G)BS}{N} \\ &\geq -\left[\mu + \frac{\kappa(1-\varepsilon G)B}{N} + \frac{\beta(1-\varepsilon G)B}{N}\right] S, \end{aligned}$$

by separating variables we have

$$\frac{dS}{S} \geq -\left[\mu + \frac{\kappa(1-\varepsilon G)B}{N} + \frac{\beta(1-\varepsilon G)B}{N}\right] dt,$$

integrating both sides, we have

$$S \geq K e^{-\left[\mu + \frac{\kappa(1-\varepsilon G)B}{N} + \frac{\beta(1-\varepsilon G)B}{N}\right] t},$$

at $t = 0$

$$S \geq K. \tag{3.10}$$

Similarly, it can be seen that all other variables $B(t), C(t), D(t), R(t) > 0$. ∎

3.2. Existence of Bandit-Free Equilibrium

In the bandit-free equilibrium, there are no bandits. As a result, every infected class will be zero and everyone in the population will be Susceptible.

$$\begin{bmatrix} S & B & C & F & D & G & N \end{bmatrix} = \begin{bmatrix} \frac{\Lambda}{\mu} & 0 & 0 & 0 & 0 & \frac{p}{q} & \frac{\Lambda}{\mu} \end{bmatrix}. \tag{3.11}$$

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3.3. Local Stability of Banditry-Free Equilibrium and Effective Reproduction number

Using the next-generation matrix developed by [21] and [22] on the model equation (2.1), we were able to establish the stability of bandits-free equilibrium. The reproduction number also denoted by R_c, is the criterion of the spread in a complete infected population. R_c can be defined as the average number of new cases of banditry influenced by typical bandits in an infected population

$$F = \begin{bmatrix} \frac{\beta(1-\varepsilon G^0)S^0}{N^0} & 0 \\ 0 & 0 \end{bmatrix}, \tag{3.12}$$

$$V = \begin{bmatrix} -(\alpha + \mu + \delta_3 G^0) & 0 \\ \alpha & -\mu \end{bmatrix}. \tag{3.13}$$

By multiplying F and V^{-1} yields to

$$FV^{-1} \quad = \quad \begin{bmatrix} \frac{\beta(q-\epsilon p)}{(\alpha q+\mu q+\delta p)} & 0 \\ 0 & 0 \end{bmatrix}. \tag{3.14}$$

The reproduction number denoted by $R_c = \rho F V^{-1}$ where ρ denotes the spectral radius. Therefore,

$$R_c = \frac{\beta(q - \varepsilon p)}{(\alpha q + \mu q + \delta p)}. \tag{3.15}$$

Therefore, by [12], [5] and [14], we determined that the model system has a bandit-free equilibrium that is locally asymptotically stable whenever $R_c < 1$. That is, banditry dies out and is unstable if and only if $R_c > 1$, meaning the bandit's activities will persist. The threshold quantity R_c is the effective reproduction number of banditry. It is the transmission of a new bandit produced by potential banditry activities.

3.4. **Existence of Local Stability of Bandits-Free Equilibrium Points**

For non-linear differential equation systems, the asymptotic stability of equilibrium is established using the Routh-Hurwitz criteria. Asymptotic stability is determined by the Routh-Hurwitz criteria, which give the necessary and sufficient conditions for all roots of the characteristic polynomial to contain negative portions. The equilibrium's local stability can be ascertained using the Jacobian matrix [20] and [15].

Let

$$m_1 = \frac{\kappa(q - \epsilon_p)B}{q}, \quad m_2 = \frac{\beta(q - \epsilon_p)B}{q}, \quad m_3 = \frac{\delta_3 p + \mu q + \alpha q}{q},$$

$$m_4 = \frac{\tau_1 q + \tau_2 p}{q}, \quad m_5 = \frac{\mu q + \sigma_1 q + \sigma_2 p}{q}.$$

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$$J(E^0) = \begin{bmatrix} -\mu & -(m_1+m_2) & 0 & 0 & 0 & 0 \\ 0 & (m_2-m_3) & 0 & 0 & 0 & 0 \\ 0 & m_1 & -(m_1+m_5) & 0 & 0 & 0 \\ 0 & 0 & m_4 & -\mu & 0 & 0 \\ 0 & \alpha & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -q \end{bmatrix}, \tag{3.16}$$

i.e.

$$|J(E^0) - \lambda I| = \begin{bmatrix} -(\mu+\lambda) & -(m_1+m_2) & 0 & 0 & 0 & 0 \\ 0 & (m_2-m_3-\lambda) & 0 & 0 & 0 & 0 \\ 0 & m_1 & -(m_1+m_5+\lambda) & 0 & 0 & 0 \\ 0 & 0 & m_4 & -(\mu+\lambda) & 0 & 0 \\ 0 & \alpha & 0 & 0 & -(\mu+\lambda) & 0 \\ 0 & 0 & 0 & 0 & 0 & -(n+\lambda) \end{bmatrix}.$$

Thus, in the absence of bandits, the bandits-free equilibrium E^0 is Locally Asymptotically Stable (LAS). Consequently, with the Maple 16 software we calculated the determinant which produces the characteristic equation for the above matrix, which is

$$(-\mu-\lambda)(m_2-m_3-\lambda)(-m_1-m_5-\lambda)(-\mu-\lambda)(-\mu-\lambda)(-\mu-\lambda) \tag{3.17}$$

simplifying
$\lambda^6 + \lambda^5(q+m_5+m_1+3\mu+m_3-m_2) + \lambda^4(3\mu^2 - m_2m_5 + m_{1q} - m_2m_1 - m_{2}q + 3\mu m_5 + 3\mu m_1 + m_5q + m_3m_1 + 3m_3\mu + m_3m_5 + m_3q + 3\mu q - 3m_2\mu) + \lambda^3(\mu^3 + 3\mu^2q - 3m_2\mu^2 + 3\mu^2m_5 + 3m_3\mu^2 + 3\mu^2m_1 + 3m_3\mu q + 3\mu m_5q + m_3m_1q + m_3m_5q - m_2m_5q - 3m_2\mu q + 3\mu m_1q - m_2m_1q + 3m_3\mu m_1 - 3m_2\mu m_5 - 3m_2\mu m_1) + \lambda^2(-m_2\mu^3 + \mu^3m_5 + \mu^3q + \mu^3m_1 + m_3\mu^3 + 3\mu^3m_1q - 3m_2\mu^2q + 3m_3\mu^2q + 3\mu^2m_5q - 3m_2\mu^2m_1 + 3m_3\mu^2m_5 + 3m_3\mu^2m_1 - 3m_2\mu^2m_5 + 3m_3\mu m_5q + 3m_3\mu m_1q - 3m_2\mu m_5q - 3m_2\mu m_1q) + \lambda(\mu^3m_5q + m_3\mu^3q - m_2\mu^3q - m_2\mu^3m_1 + m_3\mu^3m_5 + m_3\mu^3m_1 - 3m_2\mu^2m_1q + 3m_3\mu^2m_1q + 3m_3\mu^2m_5q - 3m_2\mu^2m_5q + \mu^3m_1q - m_2\mu^3m_5) + (m_3\mu^3m_1q + m_3\mu^3m_5q - m_2\mu^3m_1q - m_2\mu^3m_5q)$.
Collect the coefficient of the eigenvalues λ and characteristic give:

$$\lambda^6 + a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0, \tag{3.18}$$

where

$$\begin{aligned}
a_0 &= m_3\mu^3m_1q + m_3\mu^3m_5q - m_2\mu^3m_1q - m_2\mu^3m_5q. \\
a_1 &= \mu^3m_5q + m_3\mu^3q - m_2\mu^3q - m_2\mu^3m_1 + m_3\mu^3m_5 + m_3\mu^3m_1 - 3m_2\mu^2m_1q \\
&\quad + 3m_3\mu^2m_1q + 3m_3\mu^2m_5q - 3m_2\mu^2m_5q + \mu^3m_1q - m_2\mu^3m_5. \\
a_2 &= -m_2\mu^3 + \mu^3m_5 + \mu^3n + \mu^3m_1 + m_3\mu^3 + 3\mu^3m_1q - 3m_2\mu^2q + 3m_3\mu^2q \\
&\quad + 3\mu^2m_5q - 3m_2\mu^2m_1 + 3m_3\mu^2m_5 + 3m_3\mu^2m_1 - 3m_2\mu^2m_5 + 3m_3\mu m_5q \\
&\quad + 3m_3\mu m_1q - 3m_2\mu m_5q - 3m_2\mu m_1q. \\
a_3 &= \mu^3 + 3\mu^2q - 3m_2\mu^2 + 3\mu^2m_5 + 3m_3\mu^2 + 3\mu^2m_1 + 3m_3\mu q + 3\mu m_5q \\
&\quad + m_3m_1q + m_3m_5q - m_2m_5q - 3m_2\mu q + 3\mu m_1q - m_2m_1q + 3m_3\mu m_1 \\
&\quad - 3m_2\mu m_5 - 3m_2\mu m_1.
\end{aligned}$$

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$$a_4 = 3\mu^2 - m_2m_5 + m_1q - m_2m_1 - m_2q + 3\mu m_5 + 3\mu m_1 + m_5q + m_3m_1 + 3m_3\mu + m_3m_5 + m_3q + 3\mu q - 3m_2\mu.$$
$$a_5 = q + m_5 + m_1 + 3\mu + m_3 - m_2.$$

The Routh-Hurwitz criterion makes it evident that the eigenvalues have a negative real part, and as a result, since there are no bandits, the bandit-free equilibrium is locally asymptotically stable (LAS) [8].

3.5. Global Stability of Bandits-free Equilibrium (E^{**})

Global stability refers to the long-term performance and adaptability of the bandit model across various instances.

Theorem 3.3. *The Bandit-free equilibrium of the system of equation, (E^{**}) is globally asymptotically stable (GAS) if $R_c < 1$.*

Proof. Consider the Lyapunov function

$$F = aB + bD, \tag{3.19}$$

where a, b are constants, differentiating equation (3.19) yield

$$F' = a\left[(1-\epsilon G)\lambda S - (\delta_3 G + \mu + \alpha)B\right] + b\left[\alpha B - \mu D\right]. \tag{3.20}$$

Substituting B' and D'

$$F' = a(1-\epsilon G)\lambda S - \left[a(\delta_3 G + \mu + \alpha) - b\alpha\right]B - \mu Db. \tag{3.21}$$

We set coefficient of λS to the numerator of R_c ignoring β and substituting G, we have

$$a\frac{(q-\epsilon p)}{q} = (q - \epsilon p), \tag{3.22}$$

$$a = q. \tag{3.23}$$

We set the coefficient of D from equation (3.20) to zero, and we have

$$b = 0. \tag{3.24}$$

Substituting (3.23) and (3.24) into the equation (3.19), we have

$$F' = qB'. \tag{3.25}$$

Substituting the value of B' of the system (2.1) into the equation (3.25), we have

$$F' = q\left\{\frac{\beta(1-\epsilon G)BS}{N} - (\delta_3 G + \mu + \alpha)B\right\}, \tag{3.26}$$

$$F' = q(\delta_3 G + \mu + \alpha)B\left\{\frac{\beta(1-\epsilon G)BS}{N(\delta_3 G + \mu + \alpha)B} - 1\right\}, \tag{3.27}$$

$$F' = q(\delta_3 G + \mu + \alpha)B\left\{\frac{\beta(1-\epsilon G)S}{N(\delta_3 G + \mu + \alpha)} - 1\right\}. \tag{3.28}$$

Since,

$$N \le N^0, S \le S^0, G \le G^0. \tag{3.29}$$

Then

$$F' = (\delta_3 p + \mu q + \alpha q)B\left\{\frac{\beta(q-\epsilon p)}{(\delta_3 p + \mu q + \alpha q)} - 1\right\}. \tag{3.30}$$

i.e.

$$(\delta_3 p + \mu q + \alpha q)\, B \left\{ R_c - 1 \right\}. \tag{3.31}$$

∎

3.6. **Existence of Endemic Equilibrium State**

The endenic equilibrium state or points is a positive steady state solution at which the bandits persist in the population.

4. Local Stability of Endemic Equilibrium, (E^{**})

In mathematical modeling, the concept of local stability is crucial for understanding the dynamics of banditry spread or population interactions. An "endemic equilibrium" represents a stable state where banditry persists at a constant level or a population maintains a stable size.

To assess the local stability of an endemic equilibrium, we typically use mathematical models and techniques such as linearization and Jacobian matrices. These methods help determine whether small perturbations from the equilibrium point will lead to the system returning to that point (indicating stability) or moving away from it (indicating instability).

Understanding the local stability of an endemic equilibrium is important for predicting bandit behavior or population dynamics in specific regions and can inform public health policies, conservation efforts, and more.

$$J(E^0) = \begin{bmatrix} -\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}} + \mu\right] & -\left[(\kappa+\beta)\frac{(1-\epsilon G)S^{**}}{N^{**}}\right] & 0 & 0 & 0 & -\left[(\kappa+\beta)\frac{(\epsilon)B^{**}}{N^{**}}\right] \\ \frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}} & \frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}} - (\delta_3 G^{**} + \mu + \alpha) & 0 & 0 & 0 & \frac{\beta(\epsilon)B^{**}S^{**}}{N^{**}} - \delta_3 B^{**} \\ \frac{\kappa(1-\epsilon G^{**})B^{**}}{N^{**}} & m\frac{\kappa(1-\epsilon G^{**})S^{**}}{N^{**}} & -(\tau_1+\tau_2+\mu+\delta_1+\delta_2 G^{**}) & 0 & 0 & \frac{\kappa(\epsilon)B^{**}S^{**}}{N^{**}} - \tau_1 C^{**} - \tau_2 C^{**} \\ 0 & 0 & \tau_1 + \tau_2 G{**} & -\mu & 0 & \\ 0 & \alpha & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -q \end{bmatrix}.$$

Reducing the above matrix to an upper triangular matrix gives

$$J(E^0) = \begin{bmatrix} m1 & m2 & 0 & 0 & 0 & m3 \\ 0 & m4 & 0 & 0 & 0 & m5 \\ 0 & 0 & m6 & 0 & 0 & m7 \\ 0 & 0 & 0 & -\mu & 0 & m8 \\ 0 & 0 & 0 & 0 & -\mu & m9 \\ 0 & 0 & 0 & 0 & 0 & -q \end{bmatrix}, \tag{4.1}$$

where

$$m1 = -\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}} + \mu\right].$$

$$m2 = -\left[(\kappa+\beta)\frac{(1-\epsilon G)S^{**}}{N^{**}}\right].$$

$$m3 = -\left[(\kappa+\beta)\frac{(\epsilon)B^{**}}{N^{**}}\right].$$

$$m4 = \frac{-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]\frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}}-(\delta_3 G^{**}+\mu+\alpha)}{-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]} + \frac{-\frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}}-\left[(\kappa+\beta)\frac{(1-\epsilon G)S^{**}}{N^{**}}\right]}{-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]}.$$

$$m5 = \frac{\frac{\beta(\epsilon)B^{**}S^{**}}{N^{**}}-\delta_3 B^{**}-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]}{-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]} + \frac{-\frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}}-\left[(\kappa+\beta)\frac{(\epsilon)B^{**}}{N^{**}}\right]}{-\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]}.$$

$$m6 = -(\tau_1+\tau_2+\mu+\delta_1+\delta_2 G^{**}).$$

Theorem 4.1. *Endemic equilibrium is locally asymptotically stable if $R_c > 1$.*

Proof. It is clear that the following eigenvalues are < 0

$$\lambda_1 = -\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right] < 0,$$

$$\lambda_2 = -\frac{1}{\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]}\left[(\kappa+\beta)\frac{(1-\epsilon G)B^{**}}{N^{**}}+\mu\right]\frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}} - (\delta_3 G^{**}+\mu+\alpha) - \frac{\beta(1-\epsilon G^{**})B^{**}}{N^{**}} - \left[(\kappa+\beta)\frac{(1-\epsilon G)S^{**}}{N^{**}}\right] < 0,$$

$$\lambda_3 = -(\tau_1+\tau_2+\mu+\delta_1+\delta_2 G^{**}) < 0.$$

$$\lambda_4 = -\mu < 0,$$

$$\lambda_5 = -\mu < 0,$$

$$\lambda_6 = -q < 0.$$

It is observed that the above eighen values are less than zero. The banditry dynamics model shows that when the reproduction number (R_c) is less than one,the endemic equilibrium is locally asymptotically stable. The negative eigenvalues of the Jacobian matrix at the endemic equilibrium indicate that early efforts may be able to successfully reduce the likelihood that banditry will continue within the population. This result emphasizes the significance of maintaining (R_c) below the sustained banditry threshold, offering insightful direction for focused security measures. ∎

5. Numerical Validation

Numerical validation of analytical results is a crucial step in ensuring the accuracy and reliability of mathematical models used in scientific and engineering research. In this paper, numerical simulations were presented to monitor the dynamics of the model, for different values of reproduction number in order to verify the analytical results of both Bandits-Free and persistence.

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5.1. Parameter Estimations

In mathematical modeling, parameter values are assigned by a process that includes expert consultation, data analysis, and frequent validation to make sure the model accurately captures the phenomenon that occurs in real life. Determining these values requires data, expert opinion, and deep comprehension of the system being studied. The process may involve iterative adjustments and sensitivity analysis to gauge the impact of parameter variations on model outcomes, ultimately leading to a more reliable and informative model.

Table 2. Values for parameters of the model

S/N	Parameters	values	Source
1	Λ	200000	calculated
2	μ	0.017	NBS, 2023
3	β	$(0-1)$	Assumed
4	κ	$(0-1)$	Assumed
5	δ_1	0.1	Assumed
6	δ_2	0.01	Assumed
7	p	0.0075	Calculated
8	q	0.01	Calculated
9	$\varepsilon_1, \varepsilon_2, \tau_1, \tau_2, \alpha, \delta_3$	$(0-1)$	Controls

5.2. **Numerical Simulations**

The simulations were carried out using the parameters in Table 2, for initial conditions. Maple 16 software was used to perform the simulation.

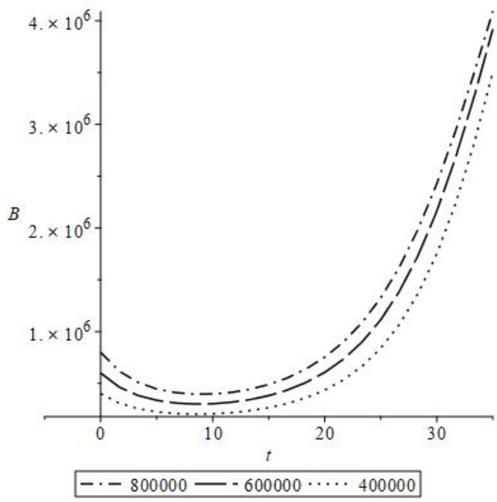


Figure 2. Total number of bandits with different initial condition: $B(1) = 800000$, $B(2) = 600000$, and $B(3) = 400000$. Control parameters used are as in Table 2, with $\varepsilon = \alpha = \delta_3 G = 0.20, \mu = 0.017$ which gives $R_c = 1.835$

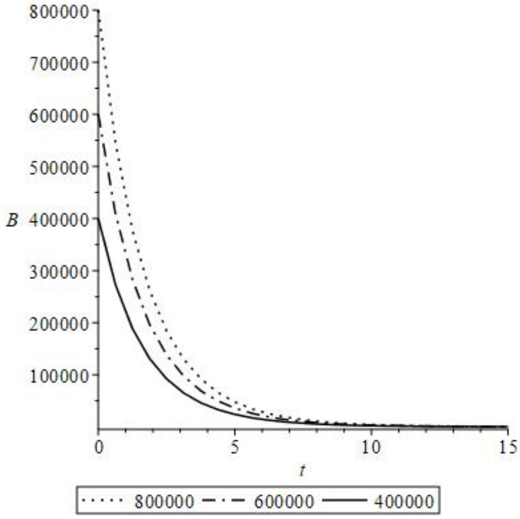
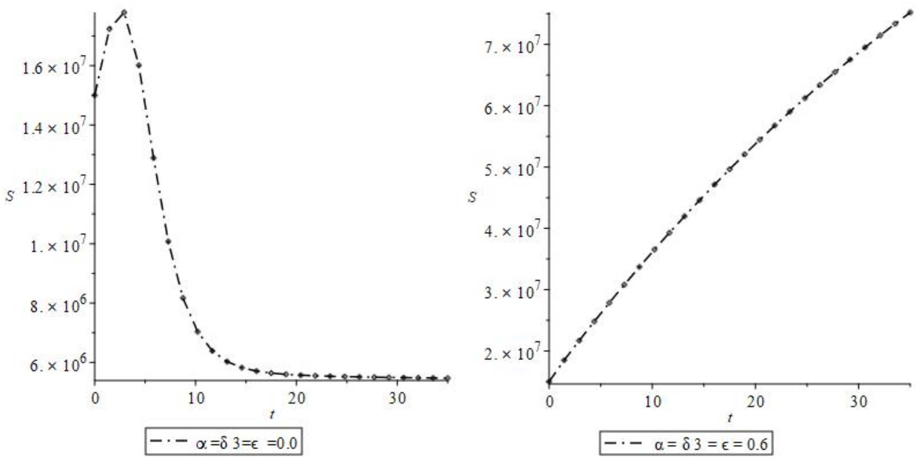
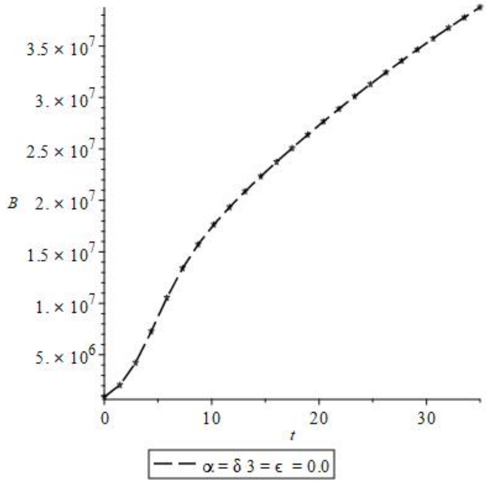


Figure 3. Total number of bandits with different initial condition: $B(1) = 800000$, $B(2) = 600000$, and $B(3) = 400000$. Control parameters used are as in Table 2, with $\varepsilon = \alpha = \delta_3 = 0.45, \mu = 0.017$ which gives $R_c = 0.650$

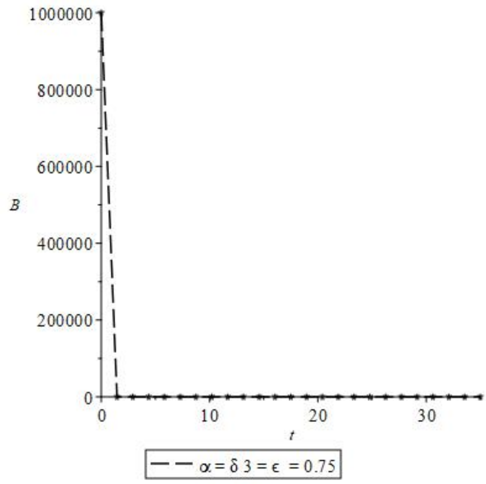


(a) graph of Susceptible without control **(b)** Graph of Susceptible with control

Figure 4. Total number of susceptible used in the above graphs was 15000000, control parameters used are as in Table 2, the first graph (a) had $\varepsilon = \alpha = \delta_3 = 0.0$ as control, while the second graph (b) had $\varepsilon = \alpha = \delta_3 = 0.60$ as control

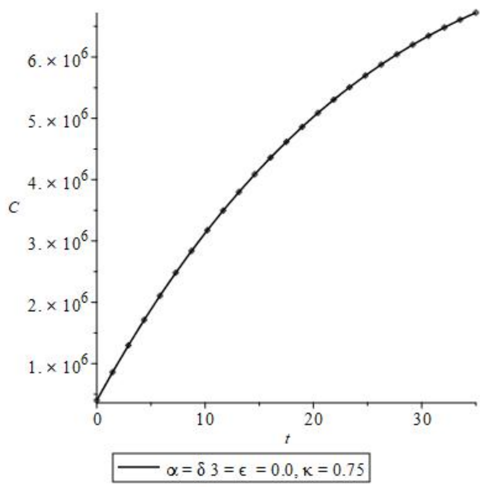


(a) graph of Bandits without control

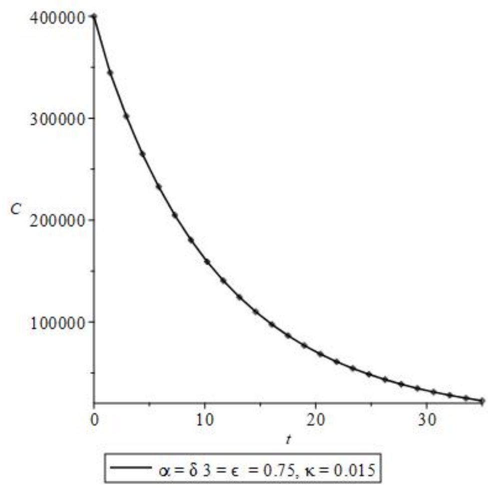


(b) Graph of Bandits with control

Figure 5. Total number of Bandits used in the above graphs was 900000, control parameters used are as in Table 2, the first graph (a) had $\varepsilon = \alpha = \delta_3 = 0.0$ as control, while the second graph (b) had $\varepsilon = \alpha = \delta_3 = 0.75$ as control



(a) graph of Captives without control



(b) Graph of Captives with control

Figure 6. Total number of Captives used in the above graphs was 400000, control parameters used are as in Table 2, the first graph (a) had $\varepsilon = \alpha = \delta_3 = 0.0, \kappa = 0.75$ as control, while the second graph (b) had $\varepsilon = \alpha = \delta_3 = 0.75\kappa = 0.015$ as control

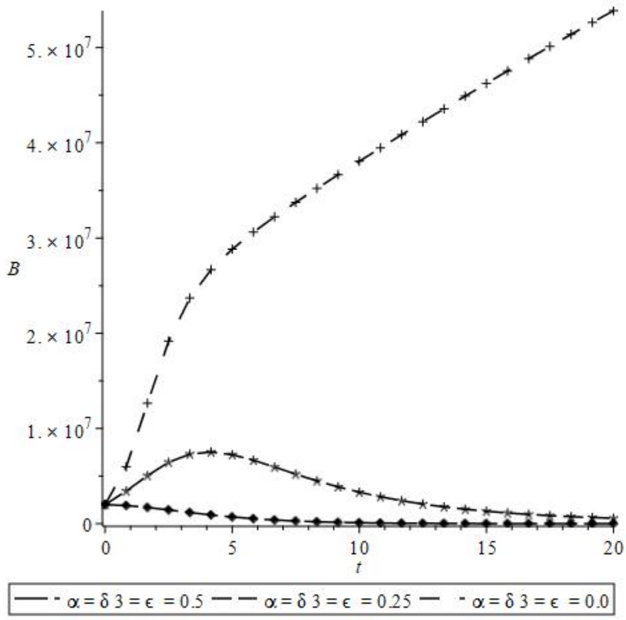


Figure 7. Comparison of the effective reproduction number of 3 different control strategy levels. Control parameters used are as in Table 2, with $\varepsilon = \alpha = \delta_3 = 0.5$, $\varepsilon = \alpha = \delta_3 = 0.25$, and $\varepsilon = \alpha = \delta_3 = 0.0$

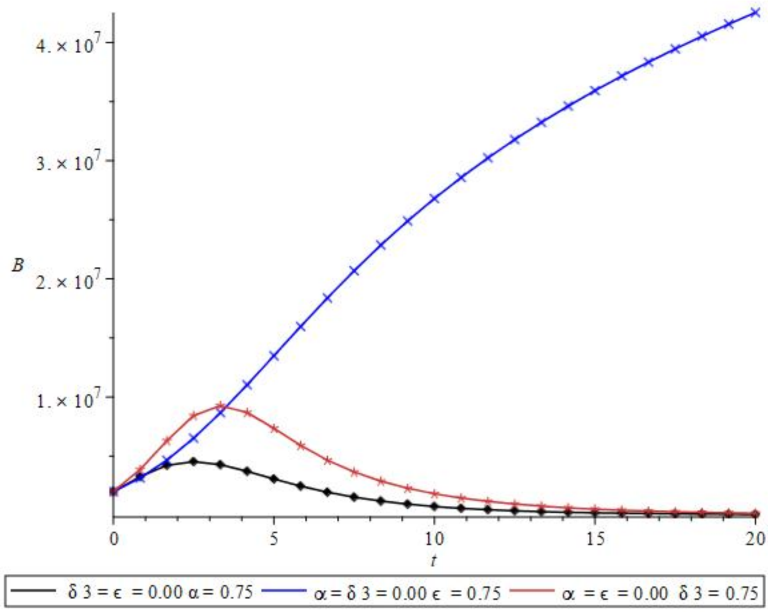


Figure 8. Comparison between detention, killing of bandits and government effort in preventing Susceptible individuals from becoming captives

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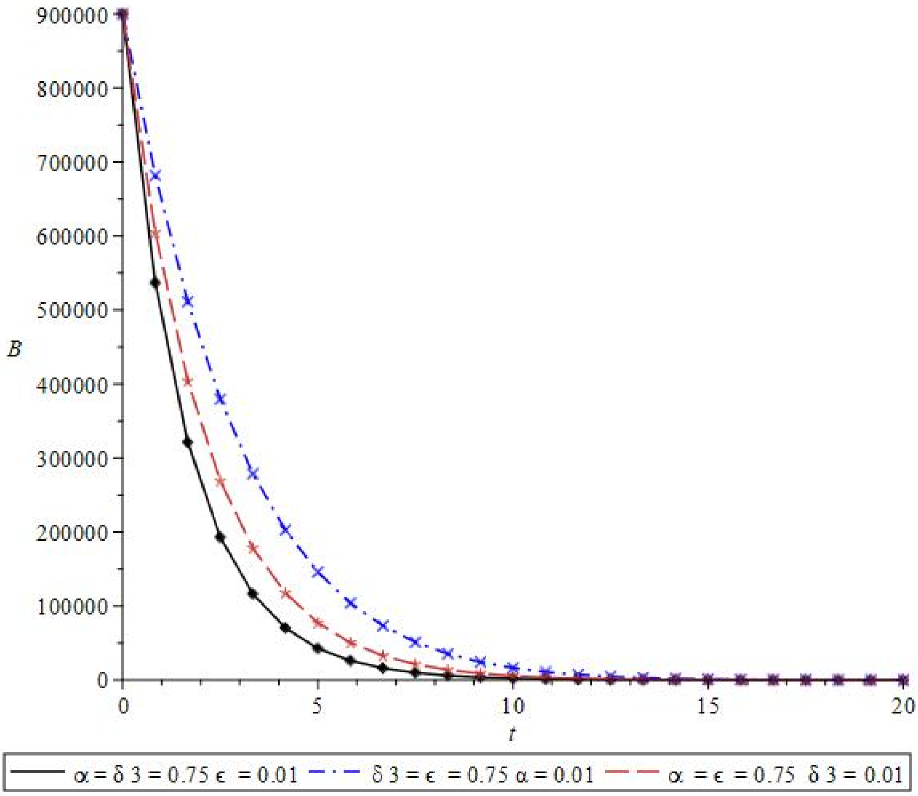

Figure 9. The graph above shows a comparison between two control strategies against one control strategies. The control parameters used are $\delta_3 = \alpha = 0.75, \varepsilon = 0.01$, $\varepsilon = \delta_3 = 0.75, \alpha = 0.01$, and $\varepsilon = \alpha = 0.75, \delta_3 = 0.00$

1. Study and investigate variational inequalities and inclusion problems on Riemannian manifolds.
2. Construct iterative algorithms for approximating variational inequalities and inclusion problems.
3. Prove the convergence theorems for the proposed algorithms for such problems.
4. Provided numerical examples for illustrate the proposed algorithms.
5. Write research papers for publication in the international journals.
6. Conclude the research project and submit the final report to the granter.

5.3. Discussion of results

Figure 2 shows the population dynamics of bandits in a bandits model, with a focus on the global asymptotic stability of the endemic equilibrium. The x-axis denotes time, while the y-axis represents the bandit population. Notably, the graph reveals that solutions with a reproduction number greater than 1 lead to the persistence of the endemic equilibrium, showcasing the crucial role this parameter plays in determining the stability of the bandit population over time. The solution after substituting the parameter values into the reproduction number was 1.638, which is greater than 1 i.e. $R_c = 1.835 > 1$.
Figure 3 provides a distinctive perspective on the banditry model, illustrating the population dynamics of bandits over time when the reproduction number is less than 1. On the x-axis, we observe time, while the y-axis represents the bandit population. In contrast to scenarios with a reproduction number greater than 1, this graph reveals a unique behavior, showing that solutions with a reproduction number less than 1 lead to a decline in the bandit population over time. The significant decline in the number of bandits shows the crucial role that the reproduction number plays in determining the stability and subsequent decline of the bandit population, providing guidance for appropriate actions.
Figure 4 showed that with zero controls, the population of Susceptible class (S) will decline with respect to time, and we also observed that with high control, the population of Susceptible class (S) will increase with respect to time.
Figure 5 displays the evaluated dynamics of our banditry model study. the first graph illustrates how the number of Bandits has increased noticeably over time due to changes in recruitment and migration rates. This trend toward growing significance emphasizes the significance of particular demands on the dynamics of banditry, which is essential to our research objectives. On the other hand, the second graph displays a drastic decreasing pattern, which is the outcome of government efforts to keep susceptible individuals from becoming banditry participants. The assessed dynamics of our study on banditry modeling are displayed in Figure 6. The number of captives has increased noticeably over time due to changes in recruitment and transition rates, as seen in the first graph. The government's efforts to keep susceptible people from becoming captives,however, are evident in the second graph, which displays a declining trend.
Figure 7 reveals that any of the three control strategies have positive effect in controlling banditry, but not all can lead to a stable bandit-free state $(R_c \leq 1)$. Whenever the controls are zero, there is no much impact on the control of banditry, and also when the controls are 0.25, there is an impact on the control of banditry. But the most stable control amongst the three controls is when the control parameters used are 0.50 as seen on the graph above.
Figure 8 is a graph of banditry against time comparing the effects on controls where we compared the effect of three (3) control parameters. the control parameters used were $\varepsilon = \delta_3 = 0.00, \alpha = 0.75$, $\delta_3 = \alpha = 0.00, \varepsilon = 0.75$ and $\varepsilon = \alpha = 0.00, \delta_3 = 0.75$. It was observed that the detention of bandits is the best strategy to curb banditry activities in Nigeria, followed by the killing of bandits and lastly the effort of the government to prevent susceptible individuals from being kidnapped.
we also observed that the detention of bandits as well as the killing of bandits have much more impact than preventing susceptible individuals from being kidnapped, the reason is that if bandits are put into detention and the killing of the bandits may put fear into others from pursuing banditry activities.
Figure 9 shows how we compare two control strategies against one, the graph shows

that killing/detention of bandits is the best way to end banditry activities, followed by killing/ability of the government to prevent susceptible individuals from being kidnapped, and lastly prevention of susceptible from been kidnapped/detention of bandits.

6. Conclusion

This research focused on banditry activities using government efforts (imprisonment or detention, killing of bandits, and preventing susceptible individuals from being kidnapped) to target and end banditry activities. Bandits have suffered multiple defeats as a result of efforts from the government through imprisonment, detention, and killing. However, the proposed model does not capture all the possible realities of the dynamics of banditry. Therefore, the results of this research can be extended by incorporating some variables and parameters such as informants, aiders, sponsors etc. to study the pattern of banditry and other social activities that pose a significant threat to the public. The numerical simulation suggests that the application of any of the controls is effective in decreasing the population profile of the bandits,but the most effective strategy is the detention of captured bandits, followed by killing of bandits and lastly the effort of government in preventing susceptible individuals from being kidnapped.

7. Recommendation

Based on the outcomes of this mathematical modeling research on banditry dynamics with control, key recommendations emerge. The adoption of improved control measures needs to be a top priority for the government, with a special emphasis on the efficiency of imprisonment and detention and focused interventions to stop banditry. Government attempts to prevent kidnappings need special focus, and specific programs that address the root causes of insecurity are necessary. To keep up with the constantly evolving banditry environment, dynamic control strategy modifications and ongoing monitoring are essential. In addition, continuous research and real-time data gathering are necessary to improve models and offer current insights for wise decision-making and long-term banditry activity control. The proposed model does not capture all the possible realities of the dynamics of banditry, like the adding of variables such as repentant bandits, who are also a good use for control of banditry activities, this is due to the fact that they have the best knowledge about banditry menace in societies. The results of this research can be extended to other fields of knowledge to study the pattern of banditry and other criminal activities in many geopolitical zones that pose a significant threat to the public.

8. Conflict of interest

On behalf of all authors, I assure you that there is no any conflict of interest.

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