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Ricci solitons and gradient Ricci solitons in Lorentzian trans-Sasakian manifolds

Kiran Kumar D. L. and Nagaraja H. G.*
Department of Mathematics, Bangalore University, Jnana Bharathi Campus, Bengaluru – 560 056, INDIA
E-mails: kirankumar250791@gmail.com; hgnraj@yahoo.com

**Corresponding author.*

Abstract The object of the present paper is to study the Ricci solitons and gradient Ricci solitons in Lorentzian trans-Sasakian manifolds. We found the condition of Ricci solitons and gradient Ricci solitons on Lorentzian trans-Sasakian manifolds to be shrinking.

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1. Introduction

Pseudo-Riemannian metrics in contact manifolds were studied first by Takahashi [14] and later by Duggal [6], Bejancu and duggal [3], Calvaruso and Perrone [5] and others. Several authors studied different contact structures on manifolds with Lorentzian metric, for example Lorentzian α-Sasakian [1] [16], Lorentzian β-Kenmotsu [2] [12], Lorentzian para-Sasakian [8] [11] and Lorentzian trans-Sasakian [13] [7] [4] manifolds.
In 1982, Hamilton [9] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. Then Ricci flow has become a powerful tool for the study of Riemannian manifolds. Shrinking Ricci solitons correspond naturally to shrinking self similar solutions to the Ricci flow. The classification of these Ricci solitons in two or three dimensions is given in [9] [10]. Extension of this classification to four and higher dimensions help us to understand the behaviour of solutions to the Ricci flow equation.
In this paper we give the characterisation of shrinking Ricci solitons in Lorentzian trans-Sasakian manifolds. After presenting basic formulae for Lorentzian trans-Sasakian manifolds in section 2, shrinking Ricci solitons in Lorentzian trans-Sasakian manifolds have

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been characterised with respect to second order parallel tensor in section 3. The shrinking gradient Ricci solitons in three dimensional Lorentzian trans sasakian manifolds are considered in section 4.

2. Preliminaries

A $(2n+1)$ dimensional differentiable manifold M is said to be a Lorentzian trans-Sasakian manifold if it admits a $(1,1)$ tensor field ϕ, a structure tensor field ξ, a 1-form η and the Lorentzian metric g which satisify

$$\phi^2 X = X + \eta(X)\xi,\ \phi\xi = 0,\ g(X,\xi) = \eta(X),\ \eta(\xi) = -1, \eta(\phi X) = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.2}$$

$$(\nabla_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \tag{2.3}$$

for all vector fields X and Y on M, where α and β are some scalar functions and such a structure is said to be the Lorentzian trans-Sasakian structure of type (α,β). We note that Lorentzian trans-Sasakian manifold of type $(0,0)$, $(0,\beta)$, $(\alpha,0)$ are the Lorentzian cosympletic, Lorentzian β-Kenmotsu and Lorentzian α- Sasakian manifolds respectively. In particular if $\alpha = 1$, $\beta = 0$ and $\alpha = 0$, $\beta = 1$, then Lorentzian trans -Sasakian manifold reduces to Lorentzian Sasakian and Lorentzian Kenmotsu manifolds respectively.
From (2.3), it follows that

$$\nabla_X \xi = -\alpha(\phi X) - \beta(X + \eta(X)\xi), \tag{2.4}$$

$$(\nabla_X \eta)Y = \alpha g(\phi X, Y) + \beta g(\phi X, \phi Y), \tag{2.5}$$

where ∇ denotes the Riemannian connection with respect to the Lorentzian metric g.
In a Lorentzian trans-Sasakian manifold [7] [13], we have

$$\begin{aligned} R(X,Y)\xi = {} & (\alpha^2+\beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi X - \eta(X)\phi Y\} \\ & + (Y\alpha)\phi X - (X\alpha)\phi Y + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y, \end{aligned} \tag{2.6}$$

$$\begin{aligned} S(X,\xi) = {} & \{2n(\alpha^2+\beta^2) - \xi\beta\}\eta(X) + (2n-1)(X\beta) - (\phi X)\alpha \\ & + \{2\alpha\beta\eta(X) + X\alpha\}\psi, \end{aligned} \tag{2.7}$$

$$\begin{aligned} Q\xi = {} & \{2n(\alpha^2+\beta^2) - \xi\beta\}\xi + (2n-1)grad\beta - \phi(grad\alpha) \\ & + \{2\alpha\beta\xi + grad\alpha\}\psi, \end{aligned} \tag{2.8}$$

$$2\alpha\beta - \xi\alpha = 0, \tag{2.9}$$

for any vector field X, Y, Z on M, where R is the Riemannian curvature tensor of type $(0,3)$, S is Ricci curvature tensor of type $(1,1)$ and Q is Ricci operator given by

$$S(X,Y) = g(QX,Y) \quad and \quad \psi = g(\phi e_i, e_i). \tag{2.10}$$

Further in a Lorentzian trans-Sasakian manifold of type (α,β) [7], we have

$$\phi(grad\alpha) = (2n-1)grad\beta. \tag{2.11}$$

For constants α and β, the equations (2.6) - (2.8) reduce to

$$R(X,Y)\xi = (\alpha^2+\beta^2)\{\eta(Y)X - \eta(X)Y\}, \tag{2.12}$$

$$S(X,\xi) = 2n(\alpha^2+\beta^2)\eta(X), \tag{2.13}$$

$$Q\xi = 2n(\alpha^2+\beta^2)\xi. \tag{2.14}$$

An important consequence of (2.4) is that ξ is a geodesic vector field. i.e. $\nabla_\xi \xi = 0$. Then for any arbitrary vector field X, we have that $d\eta(\xi, X) = 0$. The ξ-sectional curvature $K(\xi, X)$ of a Lorentzian trans-Sasakian manifold for a unit vector field X orthogonal to ξ is given by,

$$K(\xi,X) = g(R(\xi,X)\xi,X) = \alpha^2+\beta^2. \tag{2.15}$$

It follows that ξ-sectional curvature does not depend on X. If $\alpha^2+\beta^2 = 0$, then the manifold is of vanishing ξ-sectional curvature. Throughout this paper we assume that α and β are constants.

In a Riemannian manifold (M, g), g is called a Ricci soliton if

$$(L_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0, \tag{2.16}$$

where L_V is the Lie derivative along V, S is the Ricci tensor and λ is a constant. A Ricci soliton is said to be shrinking or steady or expanding according as λ is negative, zero or positive respectively. If the vector field V is gradient of some smooth function f on M, then g is called a gradient Ricci soliton and equation (2.16) assumes the form

$$\bigtriangledown \bigtriangledown f = S + \lambda g. \tag{2.17}$$

3. Second order parallel tensor and Ricci solitons in Lorentzian trans-Sasakian manifold

Definition. A tensor h of second order is said to be a parallel tensor if $\nabla h = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g.

Let h be a $(0, 2)$-type symmetric tensor field on a Lorentzian trans-Sasakian manifold M with non-vanishing ξ-sectional curvature such that $\nabla h = 0$. Then it follows that

$$\nabla^2 h(X,Y;Z,W) - \nabla^2 h(X,Y;W,Z) = 0. \tag{3.1}$$

From (3.1), we obtain the relation

$$h(R(X,Y)Z,W) - h(R(X,Y)W,Z) = 0, \tag{3.2}$$

for arbitrary vector fields X, Y, Z on M.

Substitution of $X = Z = W = \xi$ in (3.2) gives us

$$h(\xi, R(\xi,Y)\xi) = 0. \tag{3.3}$$

Using (2.12) in (3.3), we get

$$(\alpha^2+\beta^2)\{\eta(Y)h(\xi,\xi) + h(\xi,Y)\} = 0. \tag{3.4}$$

Since $(\alpha^2+\beta^2) \neq 0$, (3.4) reduces to

$$h(\xi,Y) + \eta(Y)h(\xi,\xi) = 0. \tag{3.5}$$

Taking the covariant differentiation of (3.5) with respect to X, we get

$$h(\nabla_X \xi, Y) + h(\xi, \nabla_X Y) + g(\nabla_X Y, \xi)h(\xi, \xi) + g(Y, \nabla_X \xi)h(\xi, \xi) + 2g(Y, \xi)h(\nabla_X \xi, \xi) = 0. \tag{3.6}$$

Replacing Y by $\nabla_X Y$ in (3.5), we obtain

$$h(\xi, \nabla_X Y) + g(\nabla_X Y, \xi)h(\xi, \xi) = 0. \tag{3.7}$$

In view of (3.7), it follows from (3.6) that

$$h(\nabla_X \xi, Y) + g(Y, \nabla_X \xi)h(\xi, \xi) + 2g(Y, \xi)h(\nabla_X \xi, \xi) = 0. \tag{3.8}$$

Using (2.4) in (3.8), we get

$$\{\beta g(X, Y) - \alpha g(\phi X, Y)\}h(\xi, \xi) + \{\beta h(X, Y) + \alpha h(\phi X, Y)\} = 0. \tag{3.9}$$

Replacing X by ϕX in (3.9) and then using (2.1), we obtain

$$(\alpha^2 + \beta^2)\{h(X, Y) + g(X, Y)h(\xi, \xi)\} = 0. \tag{3.10}$$

This implies

$$h(X, Y) = -g(X, Y)h(\xi, \xi). \tag{3.11}$$

Thus we state the following:

Theorem 1. *A symmetric parallel second order covariant tensor in a Lorentzian trans-Sasakian manifold of non-vanishing ξ-sectional curvature is a constant multiple of the metric tensor.*

As an immediate corollary of theorem 3.1 we have the following result.

Corollary 3.1. *A locally Ricci symmetric ($\nabla S = 0$) Lorentzian trans-Sasakian manifold of non-vanishing ξ-sectional curvature is an Einstein manifold.*

A straightforward computation gives

$$(L_\xi g)(X, Y) = -2\beta g(\phi X, \phi Y). \tag{3.12}$$

The metric g is called η-Einstein if there exists two real functions a and b such that the Ricci tensor S of g is given by

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y). \tag{3.13}$$

Let $\{e_1, e_2, e_3, ...e_{2n+1}\}$ be a local orthonormal basis of vector fields in M. Then by taking $X = Y = e_i$ in (3.13) and summing up with respect to i, we obtain

$$r = (2n + 1)a - b. \tag{3.14}$$

Again by taking $X = Y = \xi$ in (3.13) and then using (2.1) and (2.13), we get

$$-2n(\alpha^2 + \beta^2) = -a + b. \tag{3.15}$$

From (3.14) and (3.15), we obtain

$$a = \frac{r}{2n} - (\alpha^2 + \beta^2) \qquad b = \frac{r}{2n} - (2n + 1)(\alpha^2 + \beta^2). \tag{3.16}$$

Substituting the values of a and b in (3.13), we get

$$S(X, Y) = \{\frac{r}{2n} - (\alpha^2 + \beta^2)\}g(X, Y) + \{\frac{r}{2n} - (2n + 1)(\alpha^2 + \beta^2)\}\eta(X)\eta(Y). \tag{3.17}$$

Suppose

$$h(X,Y) = (L_\xi g)(X,Y) + 2S(X,Y). \tag{3.18}$$

Using (3.12) and (3.17) in (3.18), we obtain

$$\begin{aligned} h(X,Y) = &\{\frac{r}{n} - 2(\alpha^2+\beta^2) - 2\beta\}g(X,Y) \\ &+ \{\frac{r}{n} - 2(2n+1)(\alpha^2+\beta^2) - 2\beta\}\eta(X)\eta(Y). \end{aligned} \tag{3.19}$$

Taking $X = Y = \xi$ in (3.19), we get

$$h(\xi,\xi) = -4n(\alpha^2+\beta^2). \tag{3.20}$$

If (g, ξ, λ) is a Ricci soliton on a Lorentzian trans-Sasakian manifold M, then from (2.16) and (3.18), we have

$$h(X,Y) = -2\lambda g(X,Y). \tag{3.21}$$

Setting $X = Y = \xi$ in (3.21), we get

$$h(\xi,\xi) = 2\lambda. \tag{3.22}$$

Hence from (3.20) and (3.22), we have

$$\lambda = -2n(\alpha^2+\beta^2). \tag{3.23}$$

Thus we state the following:

Theorem 1. *If the tensor field $L_\xi g + 2S$ on a Lorentzian trans-Sasakian manifold with non-vanishing ξ-sectional curvature is parallel, then the Ricci soliton (g, ξ, λ) is shrinking.*

As particular cases, we state the following:

Corollary 3.2. *[1] If the tensor field $L_\xi g + 2S$ on a Lorentzian α-Sasakian manifold with $\alpha \neq 0$ is parallel, then the Ricci soliton (g, ξ, λ) is shrinking.*

Corollary 3.3. *If the tensor field $L_\xi g + 2S$ on a Lorentzian β-Kenmotsu manifold with $\beta \neq 0$ is parallel, then the Ricci soliton (g, ξ, λ) is shrinking.*

4. Gradient Ricci solitons in Lorentzian trans-Sasakian manifold

In [15] De et al. studied gradient Ricci soliton in tran-Sasakian manifold and proved that the manifold is Einstein. In this section we prove that gradient Ricci soliton in Lorentzian trans-Sasakian manifold is shrinking.
Let us consider (M, g) to be a three dimensional Lorentzian trans-Sasakian manifold with non-vanishing ξ-sectional curvature. Then we have

$$\begin{aligned} R(X,Y)Z = &g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y \\ &- \frac{r}{2}[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{4.1}$$

Setting $Y = Z = \xi$ in (4.1) and using (2.13) and (2.14), we get

$$QX = \{\frac{r}{2} - (\alpha^2+\beta^2)\}X + \{\frac{r}{2} - 3(\alpha^2+\beta^2)\}\eta(X)\xi. \tag{4.2}$$

Suppose g is a gradient Ricci soliton, then (2.17) can be written as

$$\nabla_Y Df = QY + \lambda Y, \tag{4.3}$$

for all vector fields Y on M, where D denotes the gradient operator of g.
From (4.3) it follows that

$$R(X,Y)Df = (\nabla_X Q)Y - (\nabla_Y Q)X. \tag{4.4}$$

This implies

$$g(R(\xi,Y)Df,\xi) = g((\nabla_\xi Q)Y,\xi) - g((\nabla_Y Q)\xi,\xi). \tag{4.5}$$

Now using (2.4), we have

$$g((\nabla_Y Q)\xi,\xi) = g(\nabla_Y Q\xi,\xi) - g(Q(\nabla_Y \xi),\xi) = 0. \tag{4.6}$$

Differentiating (4.2), we obtain

$$\begin{aligned}(\nabla_W Q)(X) = \frac{dr(W)}{2}\{X + \eta(X)\xi\} + (\frac{r}{2} - 3(\alpha^2+\beta^2))\\ \{-\alpha g(\phi W, X) - \beta(g(W,X) + \eta(X)\eta(W) + \eta(X)\nabla_W \xi\}.\end{aligned} \tag{4.7}$$

Replacing W by ξ in (4.7), we get

$$(\nabla_\xi Q)(X) = \frac{dr(\xi)}{2}\{X + \eta(X)\xi\}. \tag{4.8}$$

From (4.6) and (4.8), we obtain

$$g((\nabla_\xi Q)Y,\xi) - g((\nabla_Y Q)\xi,\xi) = 0. \tag{4.9}$$

Using (4.9) in (4.5), we get

$$g(R(\xi,Y)Df,\xi) = 0. \tag{4.10}$$

In view of (2.12), we have

$$g(R(\xi,Y)Df,\xi) = -(\alpha^2+\beta^2)\{g(Y,Df) + \eta(Y)\eta(Df)\}. \tag{4.11}$$

Hence from (4.10) and (4.11), it follows that

$$Df = -(\xi f)\xi, \qquad \text{since} \quad (\alpha^2+\beta^2) \neq 0. \tag{4.12}$$

Using (4.12) in (4.3), we get

$$\begin{aligned}S(X,Y) + \lambda g(X,Y) = g(\nabla_X Df, Y) = -g(\nabla_X(\xi f)\xi, Y)\\ = \alpha(\xi f)g(\phi X, Y) + \beta(\xi f)g(X,Y) + \beta(\xi f)\eta(X)\eta(Y)\\ - X(\xi f)\eta(Y).\end{aligned} \tag{4.13}$$

Setting $Y = \xi$ in (4.13), we get

$$X(\xi f) = \{2(\alpha^2+\beta^2) + \lambda\}\eta(X). \tag{4.14}$$

Interchanging X and Y in (4.13), we obtain

$$\begin{aligned}S(X,Y) + \lambda g(X,Y) = \alpha(\xi f)g(X,\phi Y) + \beta(\xi f)g(Y,X) + \beta(\xi f)\eta(Y)\eta(X)\\ - Y(\xi f)\eta(X).\end{aligned} \tag{4.15}$$

Adding (4.13) and (4.15), we get

$$\begin{aligned}2S(X,Y) + 2\lambda g(X,Y) = 2\beta(\xi f)g(X,Y) + 2\beta(\xi f)\eta(X)\eta(Y)\\ - Y(\xi f)\eta(X) - X(\xi f)\eta(Y).\end{aligned} \tag{4.16}$$

From (4.16) and (4.14), we obtain

$$\begin{aligned} S(X,Y)+\lambda g(X,Y) &= \beta(\xi f)\{g(X,Y)+\eta(X)\eta(Y)\} \\ &\quad -\{2(\alpha^2+\beta^2)+\lambda\}\eta(X)\eta(Y). \end{aligned} \tag{4.17}$$

Then using (4.3) and (4.17), we get

$$\nabla_Y Df = \beta(\xi f)\{Y+\eta(Y)\xi\} - \{2(\alpha^2+\beta^2)+\lambda\}\eta(Y)\xi. \tag{4.18}$$

Using (4.18), we compute $R(X,Y)Df$ and obtain

$$\begin{aligned} R(X,Y)Df &= \nabla_X\nabla_Y Df - \nabla_Y\nabla_X Df - \nabla_{[X,Y]}Df \\ &= \beta X(\xi f)Y - \beta Y(\xi f)X + \beta X(\xi f)\eta(Y)\xi - \beta Y(\xi f)\eta(X)\xi \\ &+ (\beta(\xi f) - 2(\alpha^2+\beta^2)-\lambda)\{(\nabla_X\eta)(Y)\xi - (\nabla_Y\eta)(X)\xi\} \\ &+ (\beta(\xi f) - 2(\alpha^2+\beta^2)-\lambda)\{\eta(Y)\nabla_X\xi - \eta(X)\nabla_Y\xi\}. \end{aligned} \tag{4.19}$$

Contracting the above with ξ, we get

$$g(R(X,Y)Df,\xi) = 2\alpha\{2(\alpha^2+\beta^2)+\lambda-\beta(\xi f)\}g(X,\phi Y) = 0. \tag{4.20}$$

Thus we have $2\alpha\{2(\alpha^2+\beta^2)+\lambda-\beta(\xi f)\}=0$.
If $\alpha=0$, then the manifold reduces to a Lorentzian β-Kenmotsu manifold.
If $2(\alpha^2+\beta^2)+\lambda-\beta(\xi f)=0$, then from (4.14), we get

$$X(\xi f) = \beta(\xi f)\eta(X). \tag{4.21}$$

By substituting (4.21) in (4.17), we obtain

$$S(X,Y)+\lambda g(X,Y) = \beta(\xi f)g(X,Y). \tag{4.22}$$

On contracting (4.22), we get

$$(\xi f) = \frac{r}{3\beta}+\frac{\lambda}{\beta}. \tag{4.23}$$

If r is a constant, then (ξf) = constant. So from (4.12), we have $Df=c\xi$, where c is a constant. Thus we can write from this equation $df=c\eta$ and its exterior derivative implies that $cd\eta=0$. Since $d\eta\neq 0$, we have $c=0$ and hence $Df=0$. This implies that f is a constant.
Consequently, equation (4.3) reduces to

$$S(X,Y) = 2(\alpha^2+\beta^2)g(X,Y). \tag{4.24}$$

This means that M is an Einstein manifold and $\lambda=-2(\alpha^2+\beta^2)$. i.e. g is shrinking.
Thus we have the following:

Theorem 1. *If a three dimensional Lorentzian trans-Sasakian manifold with non-vanishing ξ-sectional curvature admits a gradient Ricci soliton, then the manifold is either a Lorentzian β-Kenmotsu manifold or an Einstein manifold provided α and β are constants and the Ricci soliton is shrinking.*

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Website: http://tacs.kmutt.ac.th/
Email: tacs@kmutt.ac.th

