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Eco–epidemiological prey species and competitive predator species for susceptible–infected

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Abstract In this paper, we propose and analyze eco–epidemiological of prey–predator system. Here Singe prey population is taken as nature limited growth rate and interaction of both predator species, and predator population is taken as competitive of the infected–susceptible species. We propose positive and boundedness. We analyze the positive equilibrium point of trivial, disease–free, and interior equilibrium point with stability analysis. Stability analysis carried out theorem is called stable, asymptotically stable, unstable, saddle point. The dynamical behavior of this system of nonlinear differential equation both analytically and numerically is investigated from the point of view of stability analysis time series and phase portrait plot. Finally, conclusion of our results suggest that the prey–predator of the SI–type.

MSC: 26E70, 37C75, 93D05, 34G10, 34G20, 47E05, 92B05, 92D30, 92D40.

Keywords: Prey–predator model, infected–susceptible, equilibrium point, stability analysis.

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1. Introduction

Article work is very interesting and importance area for eco–epidemiological of biological dynamical system in mathematics by discussed [13]. By influencing the dynamics of phytoplankton which is the basis of all food webs in the sea, marine viruses play an even more important role, if is be-lived now that they are a key factor in global bio–geochemical cycles. Use predation may defeat spatial spread of infection by [14]. Mathematical modeling is one way to explain many of the ideas and concepts in the sciences discussed [11]. In the field of eco–epidemiological of ecology, a lot of theoretical studies were carried out since the beginning of last century to explain the interaction between the ecological communities. Particular study describes the three type of the interaction (i) prey–predator model (ii) host–parasite model (iii) infected–susceptible model between one population (prey) and the other (predator) living in a closed environment with the two populations striving for survival discuss [12], [21], [10]. A predator infected prey model with harvesting of infected prey for very interesting work follows the susceptible –infected–susceptible cycle by discuss [22]Switching from simple to complex dynamics in a predator–prey–parasite model for an interplay between infection rte and incubation delay. This type of the form is susceptible–infected and predator for Leslie–Gower model. Susceptible–infected interaction using Holling type II functional response using different ways of working very interesting. Harvesting as a disease control measure in an eco–epidemiological system, the dynamics of host –parasite predator interaction that includes non–selective prey harvesting can be described by susceptible–infected prey and predator model. This type is lot of theoretical study of the system [9]. Highly valued of the paper role of environmental disturbance in an eco– epidemiological model with disease from external source. The extended model defined by the following system of ordinary differential equation of this model is susceptible–infected prey and predator model. A model of predator–prey dynamics as modified by the action of a parasite for a predator prey system, where the prey population is divided into two independent groups [7]. A mathematical study of a predator–prey dynamics with disease in predator most models the set of nonlinear ordinary differential form prey and susceptible–infected predator model [8]. Ecological populations suffer from the various infectious diseases and these diseases have a significant role in regulating population size [6]. We use the an eco–epidemiological mathematical model with treatment (recovered) and disease infection in both prey and predator population. To do this, we will consider a plausible epidemiological context of mathematical models have become important tools for analyzing of role of standard incidence in an eco–epidemiological system with more infectious disease transmission models are descendants of the classic SIR–type [5]. Now we can discuss the effect of disease and harvesting on the dynamics of prey–predator system has been propose and studies analytically as well as numerically [2]. Explained the mathematical models which represent the dynamics of ecological systems. Now we can discuss the example of SI–type, SIS–type, SIR–type of disease are known as eco–epidemiological models. Recently, propose and analyze a prey–predator model with infectious SIS–type of disease in prey–predator population. They studied the local and global stability of the system analytically as well as numerically [3]. Now we can discuss the boundedness, existence and uniqueness of the solution [4]. Again, recently propose and analysis an eco–epidemic model with susceptible pest, infected pest and predator [1]. The dynamical behavior of the system is studied both analytically and numerically. Formulated and solved the optimal control problem and higher value of force of infection leads the system pest free. Now we can discuss the dynamics of pest and its predator model with disease in the pest and optimal use of pesticide. Before we can discuss and introduce the model and its analysis we would like to present a brief sketch of the construction of the model to indicate the biological relevance of it. To study analytically and numerically the influence of disease an an environment where two or more interacting species are present [15]-[20], we shall put emphasis on an eco–epidemiological system consisting of three species, namely (i) Prey–predator only susceptible–infected species means SI–type of single prey and predator only. (ii) Prey–predator for susceptible–infected–susceptible species means SIS–type of two prey or two predator and one prey or one predator. (iii) Prey–predator for susceptible–infected–recovered or treatment species means SIR, SIT–types of two prey or predator and one prey or predator in any where species we assume the model define. Before we can propose an eco–epidemiological model with susceptible prey and infected prey and predator is considered. Now we will present the dynamical behavior of the system is studied both analytically and numerically. A disease–free system an eco–epidemiological model further for mathematical simplicity by [23], [24]. They assumed that the mode of disease transmission follow the simple law of mass action.

2. Mathematical modeling

Organized model is as follows formulation of mathematical model. In this section some basic assumptions (i) Let x denote the population density of the prey, y denote the population density of the first susceptible predator and z denote the population density of the second infected predator respectively in time t.

$$\begin{aligned}\frac{dx}{dt} &= x\,(r - ax - c_1\,y - c_2 z)\\ \frac{dy}{dt} &= y\,(-\alpha\,z - c_4\,z + c_3 x - d_1)\\ \frac{dz}{dt} &= z\,(\alpha\,y + c_5 x - c_6 y - d_2)\end{aligned} \tag{2.1}$$

with the initial conditions

$$x(0) \geq 0, y(0) \geq 0,\ z(0) \geq 0$$

Some basic assumptions are

(1) x denote the population density of the prey, y denote the population density of the first susceptible predator and z denote the population density of the second infected predator respectively in time t.
(2) r is the intrinsic growth rate of the prey species.
(3) a is rate of competitive prey species.
(4) c_1, c_2 are the capture rate of the prey by the susceptible predator and infected predator, respectively.
(5) c_3, c_5 are the conversion factors for the susceptible predator and infected predator due to consumption of the prey.
(6) d_1, d_2 are the over crowding in the susceptible predator and infected predator respectively.
(7) c_4 is capture rate of the susceptible predator by the infected predator.
(8) c_6 is capture rate of the infected predator by the susceptible predator.
(9) α is force of infection between the infected predator and the susceptible predator.

3. Positiveness and boundedness of theorem

Theorem 3.1. *Given system of equations (2.1) is always non–negative. Then all possible solutions of the system (2.1) are positive.*

Consider, the first equations (2.1) of the system

$$\begin{aligned}\frac{dx}{x} &= (r - ax - c_1\,y - c_2 z)\,dt\\ \frac{dx}{x} &= \phi(x, y, z)dt\end{aligned} \tag{3.1}$$

where $\phi(x, y, z) = (r - ax - c_1\, y - c_2 z)$
Taking integration in the region [0, t], we get

$$x(t) = x(0)e^{\int \phi(x,y,z)dt} > 0, \ \forall\ t\ as\ x(0) \geq 0 \tag{3.2}$$

Next, consider the second set of equations (2.1) system

$$\begin{aligned} \frac{dy}{y} &= (-\alpha\, z - c_4\, z + c_3 x - d_1)\, dt \\ \frac{dy}{y} &= \varphi(x, z)dt \end{aligned} \tag{3.3}$$

where $\varphi(x, z) = (-\alpha\, z - c_4\, z + c_3 x - d_1)$
Taking integration in the region [0, t] we get

$$y(t) = y(0)e^{\int \varphi(x,z)dt} > 0, \ \forall\ t\ as\ y(0) \geq 0 \tag{3.4}$$

Next, consider the third set of equations (2.1) system we get

$$\begin{aligned} \frac{dz}{z} &= (\alpha\, y + c_5 x - c_6 y - d_2)\, dt \\ \frac{dz}{z} &= \chi(x, y)dt \end{aligned} \tag{3.5}$$

where $\chi(x, y) = z\,(\alpha\, y + c_5 x - c_6 y - d_2)$
Taking integration in the region [0,t] we get

$$z(t) = z(0)e^{\int \chi(x,y)dt} > 0, \ \forall\ t\ as\ z(0) \geq 0 \tag{3.6}$$

Hence it may be concluded that all the solutions of the system (2.1) are always positive.

Theorem 3.1. *The trajectories of the system* (2.1) *are bounded.*

Define the function $l = x+y+z$ and take its time derivative along the solution of (2.1)

$$\frac{dl}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}$$

now $\frac{dl}{dt} + \rho l \leq rx - a_1 x^2 + \rho x + \rho y + \rho z - zd_1 - zd_2$
where ρ is a positive constant for $r_1 + \rho - a_1 x \geq 0, \rho - d_1 \geq 0, \rho - d_2 \geq 0\ given\ \epsilon > 0$ there exists t_0 such that $t \geq t_0$.
$\frac{dl}{dt} + \rho l \leq m + \epsilon,\ if\ m = \min\{\frac{\rho + r_1}{a_1}, \rho - d_1, \rho - d_2\}$
Hence $\frac{d}{dt}(le^{\rho t}) \leq (m + \epsilon)e^{\rho t}$
$\Rightarrow l(t) \leq l(t_0)e^{-\rho(t-t_0)} + \frac{(m+t)}{\rho}(1 - e^{-\rho(t-t_0)})$.
Letting $t \longrightarrow 0$ then letting $\epsilon \longrightarrow 0$

$$\limsup_{t \longrightarrow \infty} l(t) \leq \frac{m}{\rho}$$

On the initial conditions, the system (2.1) is bounded.

4. Analytical solution of critical points

The equilibrium point of the parametric model (2.1) is given by steady state equations $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$. After algebraic calculation we get the trivial and non trivial equilibrium points.
(1) The trivial equilibrium point $\{x = 0, y = 0, z = 0\}$ this all prey–predator absents equilibrium always exists.
(2) Both susceptible–infected predator equilibrium point $\left\{x = \frac{r}{a}, y = 0, z = 0\right\}$ this equilibrium point prey is present, susceptible predator is absent and infected predator is absent.
(3) Infected predator–free equilibrium point $\left\{x = \frac{d_1}{c_3}, y = \frac{rc_3 - ad_1}{c_1\, c_3}, z = 0\right\}$ this equilibrium prey is present, susceptible predator is present and infected predator is absent.
(4) Susceptible predator–free equilibrium point $\left\{x = \frac{d_2}{c_5}, y = 0, z = \frac{rc_5 - ad_2}{c_2 c_5}\right\}$ this equilibrium prey is present, susceptible predator is absent and infected predator is present.
(5) Interior equilibrium point $\{x = x^*, y = y^*, z = z^*\}$
$x = \frac{\alpha^2 r - \alpha\, c_1\, d_2 + \alpha\, c_4\, r - \alpha\, rc_6 + \alpha\, c_2 d_1 - c_1\, c_4\, d_2 - c_4\, rc_6 - c_2 c_6 d_1}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - ac_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6}$,
$y = \frac{a\alpha\, d_2 + ac_4\, d_2 - \alpha\, rc_5 - c_4\, rc_5 + c_2 c_3 d_2 - c_2 c_5 d_1}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - ac_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6}$,
$z = \frac{ac_6 d_1 - a\alpha\, d_1 + \alpha\, rc_3 - c_1\, c_3 d_2 + c_1\, c_5 d_1 - rc_3 c_6}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - ac_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6}$ this equilibrium prey is present, susceptible predator is present and infected predator is present.

The system of the nonlinear differential equation (2.1) of Jacobian matrix is $J(x, y, z) =$

$$\begin{bmatrix} -2\,ax - c_1\,y - c_2 z + r & -xc_1 & -xc_2 \\ yc_3 & -\alpha\,z - c_4\,z + c_3 x - d_1 & y\,(-\alpha - c_4) \\ zc_5 & z\,(\alpha - c_6) & \alpha\,y + c_5 x - c_6 y - d_2 \end{bmatrix}$$

5. Stability analysis of the system

Theorem 5.1. *The trivial equilibrium point (0, 0, 0) of the system* (2.1) *is a saddle point.*

Proof. The Jacobian matrix of (0,0,0) is given by

$$J_1 = \begin{bmatrix} r & 0 & 0 \\ 0 & -d_1 & 0 \\ 0 & 0 & -d_2 \end{bmatrix}$$

Here the eigenvalues are $\lambda_1 = r > 0, \lambda_2 = -d_1 < 0, \lambda_3 = -d_2 < 0$. The one eigenvalue is positive and two eigenvalues is negative with the conditions $r > 0, d_1 > 0, d_2 > 0$. Therefore the equilibrium point (0,0,0) is a saddle point. ∎

Theorem 5.2. *Both Susceptible–infected predator–free equilibrium point* $\left\{x = \frac{r}{a}, y = 0, z = 0\right\}$ *of the system* (2.1) *is stable, provided that* $ad_1 > c_3 r, ad_2 > c_5 r$.

Proof. The Jacobian matrix of $\left\{x = \frac{r}{a}, y = 0, z = 0\right\}$ is

$$J_2 = \begin{bmatrix} -r & -\frac{rc_1}{a} & -\frac{rc_2}{a} \\ 0 & \frac{c_3 r}{a} - d_1 & 0 \\ 0 & 0 & \frac{c_5 r}{a} - d_2 \end{bmatrix}$$

The eigenvalues are
$\lambda_1 = -r < 0$,
$\lambda_2 = \frac{c_3 r - a d_1}{a} < 0$,
$\lambda_3 = \frac{c_5 r - a d_2}{a} < 0$.
only if $r > 0, ad_1 > c_3 r, ad_2 > c_5 r$. Therefore given equilibrium point is stable. ∎

Theorem 5.3. *The infected predator–free equilibrium point* $\left\{x = \frac{d_1}{c_3}, y = \frac{rc_3 - ad_1}{c_1\, c_3}, z = 0\right\}$ *of the system* (2.1) *is locally asymptotically stable, provided that* $a\alpha d_1 + c_1 c_3 d_2 + rc_3 c_6 > ac_6 d_1 + \alpha rc_3 + c_1 c_5 d_1$.

Proof. The Jacobian matrix is

$$J_3 = \begin{bmatrix} -2\,\frac{ad_1}{c_3} + \frac{ad_1 - c_3 r}{c_3} + r & -\frac{d_1 c_1}{c_3} & -\frac{d_1 c_2}{c_3} \\ -\frac{ad_1 - c_3 r}{c_1} & 0 & -\frac{(ad_1 - c_3 r)(-\alpha - c_4)}{c_1\, c_3} \\ 0 & 0 & -\frac{\alpha\,(ad_1 - c_3 r)}{c_1\, c_3} + \frac{c_5 d_1}{c_3} + \frac{c_6 (ad_1 - c_3 r)}{c_1\, c_3} - d_2 \end{bmatrix}$$

The corresponding eigenvalues are
$\lambda_1 = 1/2\,\frac{-ad_1 + \sqrt{a^2 {d_1}^2 + 4\,ac_3 {d_1}^2 - 4\,r{c_3}^2 d_1}}{c_3} < 0$,
$\lambda_2 = -1/2\,\frac{ad_1 + \sqrt{a^2 {d_1}^2 + 4\,ac_3 {d_1}^2 - 4\,r{c_3}^2 d_1}}{c_3} < 0$,
$\lambda_3 = \frac{ac_6 d_1 - a\alpha\, d_1 + \alpha\, rc_3 - c_1\, c_3 d_2 + c_1\, c_5 d_1 - rc_3 c_6}{c_1\, c_3} < 0$.
Here λ_1, λ_2 have negative real parts $\lambda_3 < 0$ with the conditions $a\alpha d_1 + c_1 c_3 d_2 + rc_3 c_6 > ac_6 d_1 + \alpha rc_3 + c_1 c_5 d_1$. Hence the given equilibrium point is locally asymptotically stable. ∎

Theorem 5.4. *The susceptible predator–free equilibrium point* $\left\{x = \frac{d_2}{c_5}, y = 0, z = \frac{rc_5 - ad_2}{c_2 c_5}\right\}$ *is locally asymptotically stable, provided that* $a\alpha d_2 + ac_4 d_2 + c_2 c_3 d_2 < \alpha rc_5 + c_4 rc_5 + c_2 c_5 d_1$.

Proof. The variation of the Jacobian matrix is

$$J_4 = \begin{bmatrix} -2\,\frac{ad_2}{c_5} + \frac{ad_2 - c_5 r}{c_5} + r & -\frac{d_2 c_1}{c_5} & -\frac{d_2 c_2}{c_5} \\ 0 & \frac{\alpha\,(ad_2 - c_5 r)}{c_2 c_5} + \frac{c_4\,(ad_2 - c_5 r)}{c_2 c_5} + \frac{c_3 d_2}{c_5} - d_1 & 0 \\ -\frac{ad_2 - c_5 r}{c_2} & -\frac{(ad_2 - c_5 r)(\alpha - c_6)}{c_2 c_5} & 0 \end{bmatrix}$$

The corresponding eigenvalues are
$\lambda_1 = \frac{a\alpha\, d_2 + ac_4\, d_2 - \alpha\, rc_5 - c_4\, rc_5 + c_2 c_3 d_2 - c_2 c_5 d_1}{c_2 c_5} < 0$,
$\lambda_2 = 1/2\,\frac{-ad_2 + \sqrt{a^2 {d_2}^2 + 4\,ac_5 {d_2}^2 - 4\,r{c_5}^2 d_2}}{c_5} < 0$,
$\lambda_3 = -1/2\,\frac{ad_2 + \sqrt{a^2 {d_2}^2 + 4\,ac_5 {d_2}^2 - 4\,r{c_5}^2 d_2}}{c_5} < 0$.
Here λ_1, λ_2 have negative real parts $\lambda_3 < 0$ with the conditions $a\alpha d_2 + ac_4 d_2 + c_2 c_3 d_2 < \alpha rc_5 + c_4 rc_5 + c_2 c_5 d_1$. Hence this system is locally asymptotically stable. ∎

Theorem 5.5. *The interior equilibrium point* $\{x = x^*, y = y^*, z = z^*\}$ *is locally asymptotically stable.*

Proof. The variation of the Jacobian matrix is

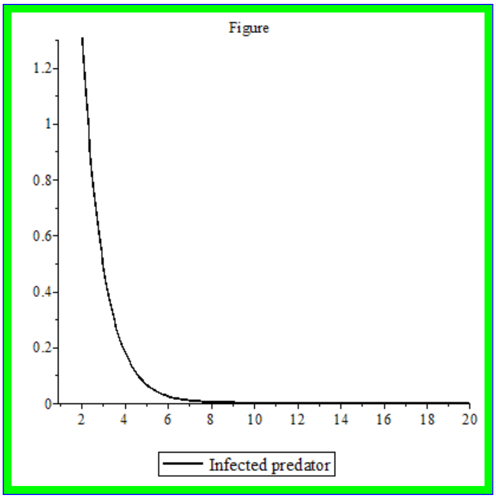

FIGURE 1. The infected predator population.

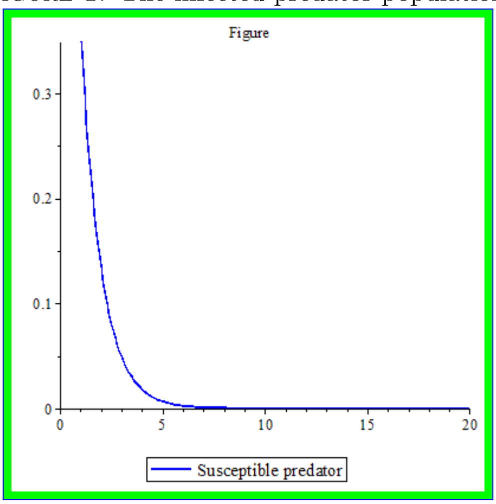

FIGURE 2. The susceptible predator population.

$$J_5 = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

where

$$m_{11} = -2\,\frac{a\left(\alpha^2 r-\alpha\, c_1\, d_2+\alpha\, c_4\, r-\alpha\, r c_6+\alpha\, c_2 d_1-c_1\, c_4\, d_2-c_4\, r c_6-c_2 c_6 d_1\right)}{a\alpha^2+a\alpha\, c_4-a\alpha\, c_6-a c_4\, c_6-\alpha\, c_1\, c_5+\alpha\, c_2 c_3-c_1\, c_4\, c_5-c_2 c_3 c_6}$$
$$-\,\frac{c_1\,(a\alpha\, d_2+a c_4\, d_2-\alpha\, r c_5-c_4\, r c_5+c_2 c_3 d_2-c_2 c_5 d_1)}{a\alpha^2+a\alpha\, c_4-a\alpha\, c_6-a c_4\, c_6-\alpha\, c_1\, c_5+\alpha\, c_2 c_3-c_1\, c_4\, c_5-c_2 c_3 c_6}$$
$$+\,\frac{c_2\,(a\alpha\, d_1-a c_6 d_1-\alpha\, r c_3+c_1\, c_3 d_2-c_1\, c_5 d_1+r c_3 c_6)}{a\alpha^2+a\alpha\, c_4-a\alpha\, c_6-a c_4\, c_6-\alpha\, c_1\, c_5+\alpha\, c_2 c_3-c_1\, c_4\, c_5-c_2 c_3 c_6}+r,$$
$$m_{12} = -\,\frac{\left(\alpha^2 r-\alpha\, c_1\, d_2+\alpha\, c_4\, r-\alpha\, r c_6+\alpha\, c_2 d_1-c_1\, c_4\, d_2-c_4\, r c_6-c_2 c_6 d_1\right)c_1}{a\alpha^2+a\alpha\, c_4-a\alpha\, c_6-a c_4\, c_6-\alpha\, c_1\, c_5+\alpha\, c_2 c_3-c_1\, c_4\, c_5-c_2 c_3 c_6},$$

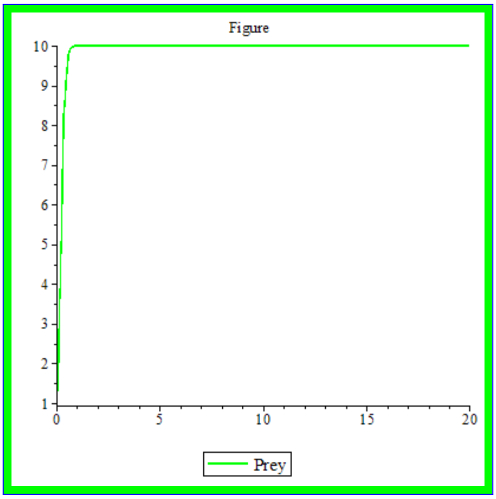

FIGURE 3. Interaction of prey population.

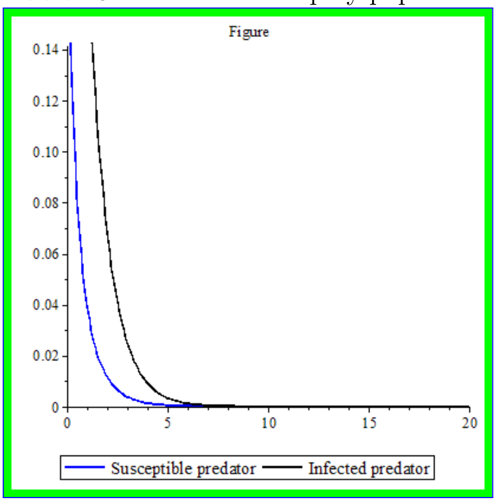

FIGURE 4. Interaction of susceptible predator and infected predator population.

$$m_{13} = -\frac{\left(\alpha^2 r - \alpha c_1 d_2 + \alpha c_4 r - \alpha r c_6 + \alpha c_2 d_1 - c_1 c_4 d_2 - c_4 r c_6 - c_2 c_6 d_1\right) c_2}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6},$$

$$m_{21} = \frac{(a\alpha d_2 + a c_4 d_2 - \alpha r c_5 - c_4 r c_5 + c_2 c_3 d_2 - c_2 c_5 d_1) c_3}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6},$$

$$m_{22} = \frac{\alpha (a\alpha d_1 - a c_6 d_1 - \alpha r c_3 + c_1 c_3 d_2 - c_1 c_5 d_1 + r c_3 c_6)}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6}$$
$$+ \frac{c_4 (a\alpha d_1 - a c_6 d_1 - \alpha r c_3 + c_1 c_3 d_2 - c_1 c_5 d_1 + r c_3 c_6)}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6}$$
$$+ \frac{c_3\left(\alpha^2 r - \alpha c_1 d_2 + \alpha c_4 r - \alpha r c_6 + \alpha c_2 d_1 - c_1 c_4 d_2 - c_4 r c_6 - c_2 c_6 d_1\right)}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6} - d_1,$$

$$m_{23} = \frac{(a\alpha d_2 + a c_4 d_2 - \alpha r c_5 - c_4 r c_5 + c_2 c_3 d_2 - c_2 c_5 d_1)(-\alpha - c_4)}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6},$$

$$m_{31} = -\frac{(a\alpha d_1 - a c_6 d_1 - \alpha r c_3 + c_1 c_3 d_2 - c_1 c_5 d_1 + r c_3 c_6) c_5}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6},$$

$$m_{32} = -\frac{(a\alpha d_1 - a c_6 d_1 - \alpha r c_3 + c_1 c_3 d_2 - c_1 c_5 d_1 + r c_3 c_6)(\alpha - c_6)}{a\alpha^2 + a\alpha c_4 - a\alpha c_6 - a c_4 c_6 - \alpha c_1 c_5 + \alpha c_2 c_3 - c_1 c_4 c_5 - c_2 c_3 c_6},$$

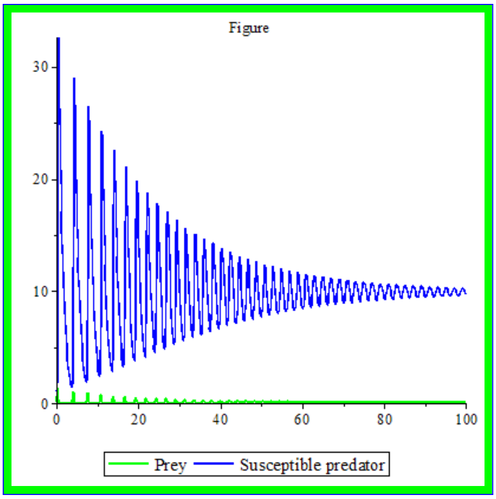

FIGURE 5. The interaction of prey and susceptible predator population.

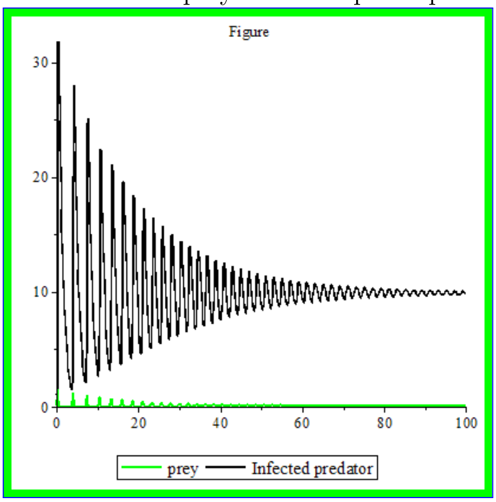

FIGURE 6. The interaction of prey and infected predator population.

$$m_{33} = \frac{\alpha\left(a\alpha\, d_2 + a c_4\, d_2 - \alpha\, r c_5 - c_4\, r c_5 + c_2 c_3 d_2 - c_2 c_5 d_1\right)}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - a c_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6}$$
$$+ \frac{c_5\left(\alpha^2 r - \alpha\, c_1\, d_2 + \alpha\, c_4\, r - \alpha\, r c_6 + \alpha\, c_2 d_1 - c_1\, c_4\, d_2 - c_4\, r c_6 - c_2 c_6 d_1\right)}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - a c_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6}$$
$$- \frac{c_6\left(a\alpha\, d_2 + a c_4\, d_2 - \alpha\, r c_5 - c_4\, r c_5 + c_2 c_3 d_2 - c_2 c_5 d_1\right)}{a\alpha^2 + a\alpha\, c_4 - a\alpha\, c_6 - a c_4\, c_6 - \alpha\, c_1\, c_5 + \alpha\, c_2 c_3 - c_1\, c_4\, c_5 - c_2 c_3 c_6} - d_2.$$

The characteristic equation is $\Lambda_1(\lambda) = B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4$
where
$B_1 = 1$,
$B_2 = -\left(m_{33} + m_{22} + m_{11}\right)$,
$B_3 = -(-m_{11}m_{22} - m_{11}m_{33} + m_{12}m_{21} + m_{13}m_{31} - m_{22}m_{33} + m_{23}m_{32})$,
$B_4 = m_{11}m_{22}m_{33} + m_{11}m_{23}m_{32} + m_{12}m_{21}m_{33} - m_{12}m_{23}m_{31} - m_{13}m_{21}m_{32} + m_{13}m_{22}m_{31}$.
By Routh Hurwitzs criterion, all the eigenvalues of J_5 have negative real parts if $(i) B_1 > 0$,

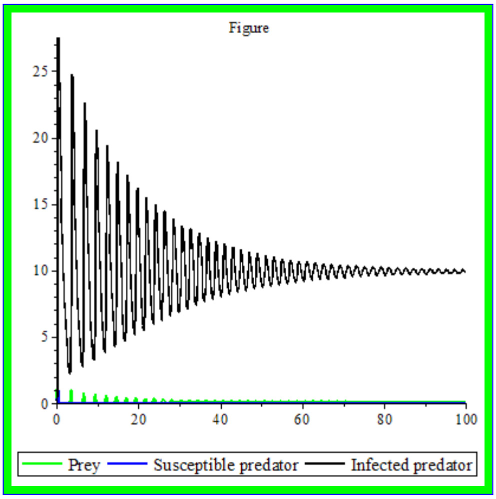

FIGURE 7. The interaction of prey and susceptible–infected predator population.

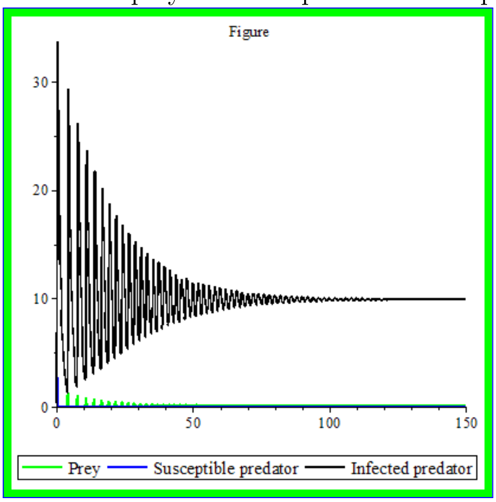

FIGURE 8. The interaction of prey and susceptible–infected predator population.

$(ii) B_3 > 0$,
$(iii) B_1B_2B_3 > B_3^2 + B_1^2B_4$.
Therefore given system of the nonlinear differential equation (2.1) is locally asymptotically stable around non–trivial equilibrium point $\{x = x^*, y = y^*, z = z^*\}$ if the conditions stated in the theorem holds. ∎

6. Numerical solution

The system of the nonlinear differential equation (2.1) for the numerical solution

(1) First we take the parameters of the system as $\rho_1 = (\alpha = 1, r = 10, a = 1, c_1 = 1, c_2 = 1, c_3 = 12, c_4 = 1, c_5 = 10, c_6 = 1, d_1 = 1, d_2 = 1)$. Then the initial conditions satisfied $(x(0) = 0, y(0) = 0, z(0) = 10)$, the infected predator

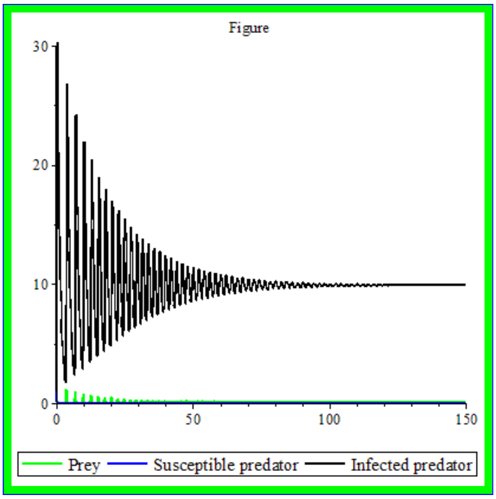

FIGURE 9. The interaction of prey and susceptible–infected predator population.

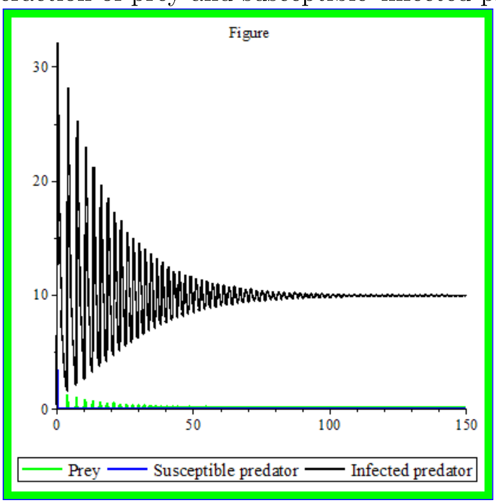

FIGURE 10. The interaction of prey and susceptible–infected predator population.

population only available (see Figure 1) is periodic at 3.67879350770605 and the infected predator population is decreasing due to absence of prey population.

(2) If we take the parameter ρ_1 of the system as mentioned above. Then the initial conditions satisfied with $(x(0) = 0, y(0) = 1, z(0) = 0)$, the susceptible predator population is available (see Figure 2) which is a periodic point at 0.367879356307219 and susceptible predator population is decreasing due to absence of prey population.

(3) If we take the parameter ρ_1 of the system as mentioned above. Then the initial conditions satisfies with $(x(0) = 1, y(0) = 0, z(0) = 0)$ the prey population (see Figure 3) is periodic at 9.99591738061742 and prey population is increasing due to absence of predator population.

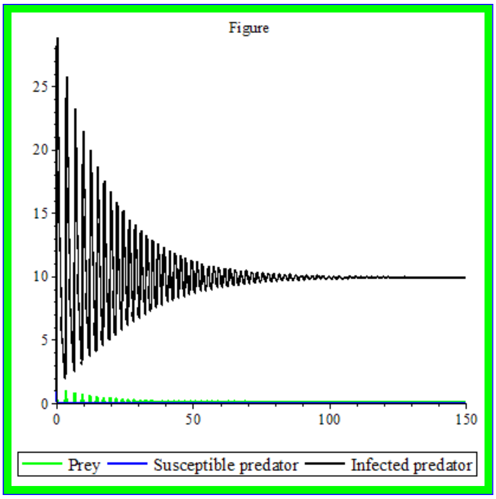

FIGURE 11. The interaction of prey and susceptible–infected predator population.

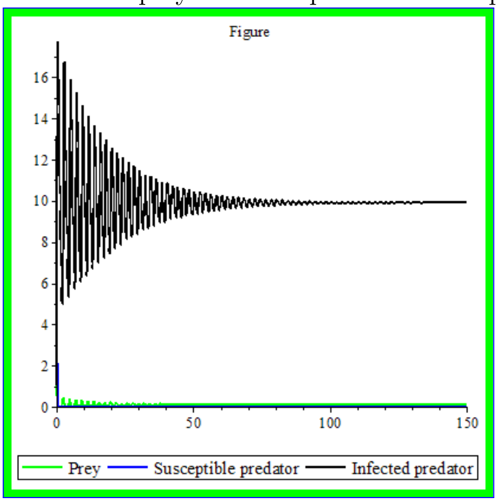

FIGURE 12. The interaction of prey and susceptible–infected predator population.

(4) Now we take the parameter ρ_1 of the system as mentioned above. Then the initial conditions satisfies with $(x(0) = 0, y(0) = 0.2, z(0) = 0.5)$, the susceptible–infected predator population (see Figure 4) is decreasing due to absence of prey population.
(5) Now we take the parameter ρ_1 of the system as mentioned above. Then the initial conditions satisfies $(x(0) = 1.0, y(0) = .1, z(0) = 0)$, from Figure 5, we can see that the interaction takes place for prey and susceptible predator species.
(6) Now we take the parameter ρ_1 of the system as mentioned above. Then the initial conditions satisfies $(x(0) = 0.1, y(0) = 0, z(0) = 1)$, from Figure 6, we can see that the interaction takes place for prey and infected predator species.

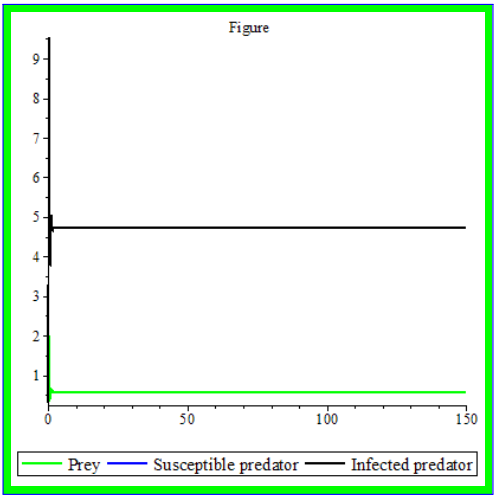

FIGURE 13. The interaction of prey and susceptible–infected predator population.

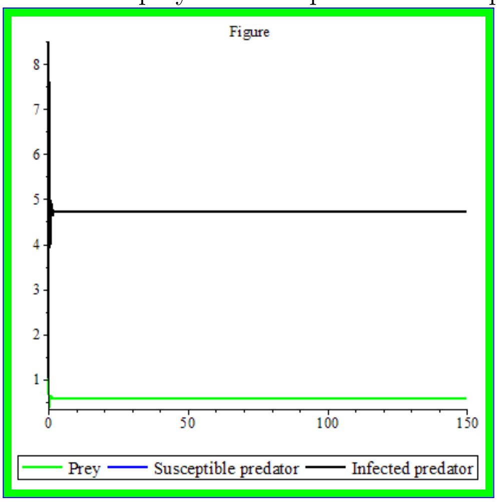

FIGURE 14. The interaction of prey and susceptible–infected predator population.

(7) Now we take the parameters of the system as ρ_1. Then the initial conditions satisfies $(x(0) = 0.15, y(0) = 0.15, z(0) = 0.15)$, from Figure 7 we can see that the interaction takes place for prey and both susceptible–infected predator species.
(8) If we take the parameters of the system as $\rho_2 = (\alpha = 1, 0.8, 0.2, 0, r = 5, a = 1, c_1 = 1, c_2 = 1, c_3 = 5, c_4 = 1, c_5 = 5, c_6 = 1, d_1 = 1, d_2 = 1)$. Then the initial conditions satisfies with $[x(0) = .3, y(0) = .3, z(0) = .3], [x(0) = 1, y(0) = 1, z(0) = 1]$ for both susceptible–infected predator and prey population of the time series (see in figure 8, 9, 10, 11, 12, 13, 14.)
(9) If we take the parameters of the system as $\rho_3 = (\alpha = 0, 0.5, 1, r = 5, a = 1, c_1 = 1, c_2 = 1, c_3 = 5, c_4 = 1, c_5 = 5, c_6 = 1, d_1 = 1, d_2 = 1)$. Then the initial conditions satisfies with $[x(0) = .25, y(0) = .5, z(0) = 2.5], [x(0) = .5, y(0) =$

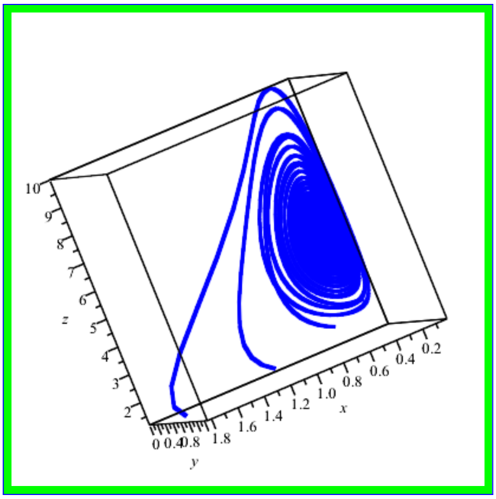

FIGURE 15. The phase plot is asymptotically stable at $\alpha = 0$.

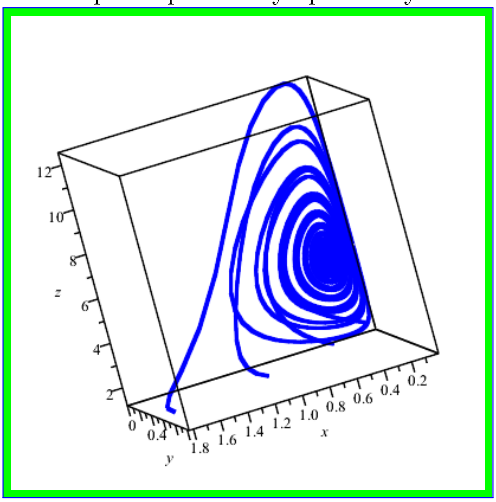

FIGURE 16. The phase plot is asymptotically stable at $\alpha = 0.5$.

$.5, z(0) = 2], [x(0) = 1, y(0) = .5, z(0) = 1.5], [x(0) = 1.7, y(0) = .5, z(0) = 1.2]$, here both susceptible–infected predator and prey population of the phase plot as shown in figure 15,16,17.

7. Discussion and conclusions

An eco–epidemiological model consisting of prey–predator model with SI-type of disease in prey population and susceptible–infected predator population was proposed and analyzed. It is observed that the system is positive and bounded has at most five trivial, disease–free, non–trivial equilibrium points. It is observed that the diseased predator population decreases due to absence of prey as shown in figures 1, 2, 4. and with out predator population, the prey population increases as shown in figure due to 3, also we

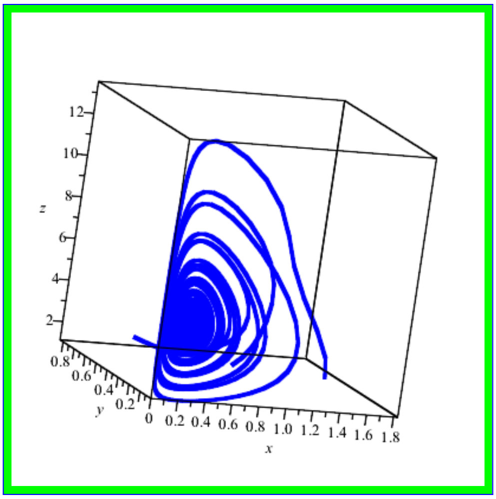

FIGURE 17. The phase plot is asymptotically stable at $\alpha = 1$.

can see that the interaction takes place for prey and susceptible–infected predator species in figures 7 to 14.

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