Bangmod Int. **J.** **M**ath. **&** **C**omp. **S**ci.
Vol. 5, No. 1-2, 2019; Pages 27 - 36
http://bangmod-jmcs.kmutt.ac.th/
ISSN : 2408-154X

Some coincidence and common fixed point results in non archimedean Menger PM-Spaces

Rajinder Sharma* and Deepti Thakur
College of Applied Sciences;
Mathematical Sciences Unit, Sohar, Oman
E-mails: rajind.math@gmail.com; thakurdeepti@yahoo.com

**Corresponding author.*

Abstract The objective of this study is to establish some common fixed point theorems for two pairs of self maps by using the notion of weakly sub-sequentially continuous (wsc) and compatibility of type (E) maps in a non Archimedean Menger PM space (briefly N.A. Menger PM-space). We improve the results of Chouhan et.al.[7, Theorem 3.2], Rao and Ramudu[22, Theorem 14], Khan and Sumitra [16, Theorem 2] and Kutukcu and Sharma [17, Theorem 1]. We also present few examples in order to validate the claim.

MSC: 47H10; 54H25

Keywords: fixed point , non Archimedean Menger PM-Space, weakly subsequentially continuous maps (wsc).

Submission date: 19 October 2019 / Acceptance date: 9 December 2019 / Available online 31 December 2019

1. Introduction

Last more than four decades witnessed an exponential growth in the field of fixed point theory and its applications to study the existence, establishment and uniqueness of common fixed points in distinct metric structured spaces especially where the uncertain situations arises such as probabilistic metric spaces. The study of these spaces are of utmost importance chiefly in the settings where the distance between the two points are unknown but the probabilities of the possible values of the distances are known. Menger [18] devised the concept of probabilistic metric space and Schweizer and Sklar [23, 25] explored the study further on statistical metric spaces. Working on the same line, Sehgal and Bharucha-Reid [24] studied some fixed points of contraction mappings on probabilistic metric spaces. Istratescu and Crivat [13] generated some fixed point results on non-Archimedean PM-spaces. Istratescu [11,12] goes one step ahead by generalizing the results of Sehgal and Bharucha [24] to N. A. Menger PM-space where as Achari [1]

Published by Theoretical and Computational Science Center (TaCS), King Mongkut's University of Technology Thonburi (KMUTT)

improved and generalized the results of Istratescu [11,12] by establishing common fixed point theorems for qausi-contraction maps in N.A. PM-space. Hadzic [10] brought forward the legacy by improving the results of Istratescu[11] in Non-Archimedean Menger spaces. Thereafter, many researchers [32, 33, 15, 9] involved in the field of fixed point theory acclaimed several common fixed point theorems using different contractive conditions and concepts viz. weakly commuting mappings, compatible mappings, compatible mappings of type A, weakly compatible mappings etc. in N.A. Menger PM-spaces. Kutukcu and Sharma [17] coined the concept of compatible mappings of type $(A-1)$ and type $(A-2)$ in Non-Archimedean Menger space and used certain conditions in order to claim their equivalency to compatible mappings. It is worth mentioning that the common fixed point results established by Bouhadjera and Thobie [4] by using the notion of sub-compatible and subsequential maps are weaker than that of occasoinally weakly compatible maps introduced by Thagafi and Sahzad [34] and reciprocal continuity by Pant [20] respectively. Beloul [2] and Bouhadjera [3] used pairs of weakly subsequentially continuous mappings to establish fixed point results in metric spaces. Chauhan et.al. [7] gave some fixed point theorems for two self pairs by using the notions of compatible and subsequentially continuity (alternatively subcompatibility and reciprocal continuity) in N.A. Menger PM-spaces. We refer ([5], [8], [9], [14], [21], [26],[28],[29],[30],[31]) for more details.
The present study deals with the notion of weakly subsequentially continuous and compatibility of type (E) maps in non Archimedean Menger PM-space and to establish some coincidence and fixed point results for the same. We also provide few examples to validate the claim.

2. Preliminaries

Definition 2.1. [11,13] Let X be any nonempty set and D be the set of all left-continuous distribution functions. An ordered pair (X, F) is called a Non Archimedean probabilistic metric space (briefly, a N.A. PM-space) if F is a mapping from $X \times X$ into mapping D satisfying the following conditions (we shall denote the distribution function $F(x, y)$ by $F_{x,y}, \forall x, y \in X$) :

$$F_{x,y}(t) = 1, \forall t > 0 \text{ if and only if } x = y \tag{2.1}$$

$$F_{x,y}(0) = 0, \ \forall x, y \in X \tag{2.2}$$

$$F_{x,y}(t) = F_{y,x}(t), \ \forall x, y \in X \tag{2.3}$$

$$F_{x,y}(t_1) = 1 \text{ and } F_{y,z}(t_2) = 1, \text{then } F_{x,z}\{\max(t_1, t_2)\} = 1, \ \forall x, y, z \in X \tag{2.4}$$

Definition 2.2. [23] A $t-$ norm is a function $\Delta : [0, 1]\times[0, 1] \to [0, 1]$ which is associative ,commutative, non-decreasing in each coordinate and $\Delta(a, 1) = a, \forall a \in [0, 1]$.

Definition 2.3. [11,13] A Non Archimedean Menger PM-space is an ordered triplet (X, F, Δ) ,where Δ is a $t-$ norm and (X, F) is a non Archimedean PM-space satisfying the following condition:

$$F_{(x,z)}(\max\{t_1, t_2\}) \geq \Delta(F_{(x,y)}(t_1), F_{(y,z)}(t_2)), \forall x, y, z \in X \text{ and } t_1, t_2 \geq 0. \tag{2.5}$$

Definition 2.4. [9] A Non Archimedean Menger PM-space (X, F, Δ), is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that $g(F_{(x,z)}(t)) \leq g(F_{(x,y)}(t)) + g(F_{(y,z)}(t))$, $\forall x, y, z \in X$ and $t \geq 0$, where $\Omega = \{g | g : [0, 1] \to [0, \infty)$ is continuous, strictly decreasing with $g(1) = 0$ and $g(0) < \infty\}$.

Definition 2.5. [9] A N.A. Menger PM-space (X, F, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that $g(\Delta(s, t)) \leq g(s) + g(t)$ for all $s, t \in (0, 1)$.

Remark 1.1. [9]

$$\begin{aligned} &A\,Non\,Archimedean\,Menger\,PM-space\,(X, F, \Delta)\,is\,of\,type\,(D)_g,\\ &then\,it\,is\,of\,type\,(C)_g. \end{aligned} \tag{2.6}$$

If (X, F, Δ) is a N. A. Menger PM-space and $\Delta \geq \Delta_m$, where $\Delta_m(s, t) = \max\{s + t - 1, 1\}$, then (X, F, Δ) is of type $(D)_g$ for $g \in \Omega$ defined by $g(t) = 1 - t$. Throughout this paper, let (X, F, Δ) be a Non Archimedean Menger PM-space of type $(D)_g$ with a continuous strictly increasing $t-$ norm Δ.
Let $\phi : [0, \infty) \to [0, \infty)$ be a function satisfying the following condition (Φ):
(Φ) ϕ is a upper semi continuous from the right and $\phi(t) < t$ for all $t > 0$.

Lemma 2.6. [9] *If a function $\phi : [0, \infty) \to [0, \infty)$ satisfies the condition (Φ) , then we have*

$$For\,all\,t \geq 0,\ \lim_{n\to\infty} \phi^n(t) = 0,\ where\,\phi^n(t)\,is\,the\,nth\,iteration\,of\,t. \tag{2.7}$$

$$If\,\{t_n\}\,is\,a\,non-decreasing\,sequence\,of\,real\,numbers\,and\,\{t_{n+1}\} \leq \phi(t_n), \tag{2.8}$$

$n = 1, 2, ...,$ then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t \geq 0$, then $t = 0$.
Singh and Mahendra [31] introduced the notion of compatibility of type (E), in the setting of the N.A. Menger PM-spaces, it becomes

Definition 2.7. Two self maps A and S on a non Archimedean Menger PM-space (X, F, Δ) are said to be compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ and $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Definition 2.8. Two self maps A and S on a non Archimedean Menger PM-space (X, F, Δ) are said to be A-compatible of type (E), if $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$ for some $z \in X$. Pair A and S are said to be S-compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ for some $z \in X$.

Remark 1.2.[2] It is also interesting to see that if A and S are compatible of type (E), then they are A-Compatible and S-Compatible of type (E), but the converse is not true (see example 1 [2]).

Motivated from Beloul [2] and Bouhadjera[3], we redefine the following in the setting of a N.A Menger PM- space.

Definition 2.9. A pair of self mappings $\{A, s\}$ defined on a N.A. Menger PM-space is said to be weakly subsequentially continuous (in short wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$; either $\lim_{n\to\infty} ASx_n = Az$ or $\lim_{n\to\infty} SAx_n = Sz$.

Definition 2.10. A pair of self mappings $\{A, S\}$ defined on a N.A. Menger PM-space (X, F, Δ) is said to be:
a) S subsequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} SAx_n = Sz$.
b) A sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$.

3. Main Results

The following is our main result:

Theorem 3.1. *Let A, B, S and T be four self mappings of a non Archimedean Menger PM- space (X, F, Δ). If the pairs (A, S) and (B, T) are weakly subsequentially continuous and compatible of type (E), then*
(i) A and S have a coincidence point,
(ii) B and T have a coincidence point.
Further, if for all $x, y \in X$ and $t > 0$, we have:

$$g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(By, Ty, t)), \quad (3.1)$$

$\frac{1}{2}(g(F(Sx, By, t)) + g(F(Ty, Ax, t)))\}]$,
where $\phi \in \Phi$ such that $\phi : [0, \infty) \to [0, \infty)$ and $g \in \Omega$.Then A, B, S and T have a unique common fixed point in X.

Proof. Since the pair (A, S) is weakly subsequentially continuous (briefly wsc) and compatible of type (E), therefore there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some$z \in X$ and $\lim_{n\to\infty} ASx_n = Az$. The compatibility of type (E) implies that
$\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$ and $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$. Therefore $Az = Sz$, whereas in respect of the pair (B, T) being weakly subsequentially continuous (briefly wsc), there exists a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$, for some$w \in X$ and $\lim_{n\to\infty} BTy_n = Bw$. The pair $\{B, T\}$ is compatible of type (E), therefore $\lim_{n\to\infty} B^2y_n = \lim_{n\to\infty} BTy_n = Tw$ and $\lim_{n\to\infty} T^2y_n = \lim_{n\to\infty} TBy_n = Bw$, for some $w \in X$, then $Bw = Tw$.

Hence z is a coincidence point of the pair (A, S) whereas w is a coincidence point of the pair (B, T). Now we prove that $z = w$. By putting $x = x_n$ and $y = y_n$ in inequality (3.1.), we have

$$g(F(Ax_n, By_n, t)) \leq \phi[\max\{g(F(Sx_n, Ty_n, t)), g(F(Ax_n, Sx_n, t)),$$
$$g(F(By_n, Ty_n, t)), \tfrac{1}{2}(g(F(Sx_n, By_n, t)) + g(F(Ty_n, Ax_n, t)))\}].$$

Taking the limit as $n \to \infty$, we get

$$g(F(z, w, t))$$
$$\leq \phi[\max\{g(F(z, w, t)), g(F(z, z, t)), g(F(w, w, t)), \tfrac{1}{2}(g(F(z, w, t)) + g(F(w, z, t))\}]$$
$$g(F(z, w, t)) \leq \phi[\max\{g(F(z, w, t)), 0, 0, g(F(z, w, t))\}]$$

i.e. $g(F(z, w, t)) \leq \phi[g(F(z, w, t))] < g(F(z, w, t))$, a contradiction. Thus, we have $z = w$.
Now we prove that $Az = z$. By putting $x = z$ and $y = y_n$ in the inequality (3.1.), we get
$g(F(Az, By_n, t)) \leq \phi[\max\{g(F(Sz, Ty_n, t)), g(F(Az, Sz, t)), g(F(By_n, Ty_n, t)),$

$\frac{1}{2}(g(F(Sz, By_n, t)) + g(F(Ty_n, Az, t)))\}]$
Taking the limit as $n \to \infty$, we get
$g(F(Az, w, t)) \leq \phi[\max\{g(F(Sz, w, t)), g(F(Az, Sz, t)), g(F(w, w, t)),$
$\frac{1}{2}(g(F(Sz, w, t)) + g(F(w, Az, t)))\}]$
$g(F(Az, w, t)) \leq \phi[\max\{g(F(Az, w, t)), 0, 0, g(F(Az, w, t))\}]$
$g(F(Az, w, t)) \leq \phi[g(F(Az, w, t))] < g(F(Az, w, t))$ which yields $Az = w$. Since $Az = Sz$. Therefore $Az = Sz = w = z$.
Now we prove that $Bz = z$. By putting $x = \{x_n\}$ and $y = z$ in the inequality (3.1.), we get
$g(F(Ax_n, Bz, t)) \leq \phi[\max\{g(F(Sx_n, Tz, t)), g(F(Ax_n, Sx_n, t)), g(F(Bz, Tz, t)),$
$\frac{1}{2}(g(F(Sx_n, Bz, t)) + g(F(Tz, Ax_n, t)))\}]$
Taking the limit as $n \to \infty$, we get
$g(F(z, Bz, t)) \leq \phi[\max\{g(F(z, Tz, t)), g(F(z, z, t)), g(F(Bz, Tz, t)),$
$\frac{1}{2}(g(F(z, Bz, t)) + g(F(Tz, z, t)))\}]$
$g(F(z, Bz, t)) \leq \phi[\max\{g(F(z, Tz, t)), 0, 0, g(F(z, Bz, t))\}]$
$g(F(z, Bz, t)) \leq \phi[g(F(z, Bz, t))] < g(F(z, Bz, t))$, which yields $Bz = z$. Since $Bz = Tz$. Therefore, $Bz = Tz = z$. Therefore in all $z = Az = Bz = Sz = Tz$, i.e. z is a common fixed point of A, B, S and T. The uniqueness of common fixed point is an easy consequence of inequality (3.1.).

∎

If we put $A = B$ in Theorem 3.1 we have the following corollary for three mappings:

Corollary 3.2. *Let A, S and T be three self maps of a Non Archimedean Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$g(F(Ax, Ay, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(Ay, Ty, t)), \quad (3.2)$$

$\frac{1}{2}(g(F(Sx, Ay, t)) + g(F(Ty, Ax, t)))\}]$
where $\phi \in \Phi$ such that $\phi : [0, \infty) \to [0, \infty)$ and $g \in \Omega$. If the pairs $\{A, S\}$ and (A, T) are weakly sub sequentially continuous and compatible of type (E), then A, S and T have a unique common fixed point in X.

Alternatively, if we set $S = T$ in Theorem 3.1, we'll have the following corollary for three self mappings:

Corollary 3.3. *Let A, B and S be three self maps of a Non Archimedean Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(By, Sy, t)), \quad (3.3)$$

$\frac{1}{2}(g(F(Sx, By, t)) + g(F(Sy, Ax, t)))\}]$
where $\phi \in \Phi$ such that $\phi : [0, \infty) \to [0, \infty)$ and $g \in \Omega$. If the pairs (A, S) and (B, S) are weakly sub sequentially continuous and compatible of type (E), then A, B and S have a unique common fixed point in X.

If we put $S = T$ in corollary 3.1 , we have the following result for two self mappings:

Corollary 3.4. *Let A and S be two self maps of a Non Archimedean Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$g(F(Ax, Ay, t)) \leq \phi[\max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(Ay, Sy, t)), \quad (3.4)$$

$\frac{1}{2}(g(F(Sx, Ay, t)) + g(F(Sy, Ax, t)))\}]$
where $\phi \in \Phi$ such that $\phi : [0, \infty) \to [0, \infty)$ and $g \in \Omega$. If the pair (A, S) is weakly subsequentially continuous and compatible of type (E), then A and S have a unique common fixed point in X.

Remark 3.1. The conclusions of Theorem 3.1 remain true if we replace the ineqaulity (3.1) either with the inequality used in [35] or by any one of the following:

$$g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(By, Ty, t)), \quad (3.5)$$

$g(F(Sx, By, t))\}]$,
Or,

$$g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(By, Ty, t)), \quad (3.6)$$

Or,

$$g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Ty, t)) + g(F(Ax, Sx, t)) + g(F(By, Ty, t)), \quad (3.7)$$

$g(F(Sx, Ax, t)) + g(F(Sx, By, t)), g(F(Ax, Ty, t)) + g(F(Ty, By, t))\}]$,
where $\phi \in \Phi$ such that $\phi : [0, \infty) \to [0, \infty)$ and $g \in \Omega$. If the pairs (A, S) and (B, T) are weakly subsequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point in X.

Remark 3.5. The results similar to Corollaries 3.1, 3.2 and 3.3 can also be obtained in the repect of inequalities 3.5, 3.6 and 3.7.

Remark 3.6. Theorem 3.1 (also in view of remark 3.1) improve the results of chouhan et. al.[7, Theorem 3.2], Rao and Ramudu[22 , Theorem 14], Khan and Sumitra [16, Theorem 2] and Kutukcu and Sharma [17, Theorem 1].

Theorem 3.7. *Let A, B,S and T be four self maps of a Non Archimedean Menger PM-space (X, F, Δ) satisfying (3.1). Assume that*
(i) the pair (A, S) is A-compatible of type (E) and A-subsequentially continuous.
(ii) the pair (B, T) is B-compatible of type (E) and B-subsequentially continuous.
Then A, B, S and T have a unique common fixed point in X.

Proof. The proof is obvious as on the lines of theorem 3.1. ∎

Remark 3.8. The conclusions of Theorem 3.2 remain true in view of Remark 3.1.

Remark 3.9. The results similar to Corollaries 3.1, 3.2 and 3.3 can also be obtained in the respect of Theorem 3.2 and Remark 3.4.

Theorem 3.10. *Let A, B,S and T be four self maps of a Non Archimedean Menger PM-space (X, F, Δ) satisfying (3.1). Assume that*
(i) the pair (A, S) is S-compatible of type (E) and S-subsequentially continuous.
(ii) the pair (B, T) is T-compatible of type (E) and T-subsequentially continuous.
Then A, B, S and T have a unique common fixed point in X.

Proof. The proof is obvious as on the lines of theorem 3.1. ∎

Remark 3.11. The conclusions of Theorem 3.3 remain true in view of Remark 3.1.

Remark 3.12. The results similar to Corollaries 3.1, 3.2 and 3.3 can also be obtained in the respect of Theorem 3.3 and Remark 3.6.

Example 3.13. Let $X = [0, \infty)$ with the usual metric d and (X, F, Δ) be the induced Non Archimedean Menger PM-space with $g(t) = 1 - t, \forall t > 0$ and $F(x, y, t) = H(t - d(x, y)), \forall x, y \in X, \forall t > 0$ and $\Delta(a, b) = \min\{a, b\}, \forall a, b \in [0, 1]$. Set $A = B$ and $S = T$. Define the self mappings A and S as follows:

$$A(X) = \begin{cases} x, & \text{if } x \in [0, 1]; \\ x - 1, & \text{if } x \in (1, \infty). \end{cases}$$

$$S(X) = \begin{cases} \frac{x+1}{2}, & \text{if } x \in [0, 1]; \\ \frac{x-1}{2}, & \text{if } x \in (1, \infty). \end{cases}$$

Let us consider a sequence $\{x_n\} = \{1 - \frac{1}{n}\}_{n \in N}$ in X. Then
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty}(1 - \frac{1}{n}) = 1 = \lim_{n\to\infty} Sx_n$. Also,
$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} AS(1 - \frac{1}{n}) = \lim_{n\to\infty} A(\frac{1-\frac{1}{n}+1}{2}) = 1 = A(1)$.

$\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} A(1 - \frac{1}{n}) = 1 = S(1)$.
$\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} S(1 - \frac{1}{2n}) = 1 = A(1)$.
Therefore, (A, S) is weak subsequentially continuous and compatible of type (E).
Next, consider another sequence $\{x_n\} = \{1 + \frac{1}{n}\}_{n \in N}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty}(\frac{1}{n}) = 0 = \lim_{n\to\infty} Sx_n$.

Also,

$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} AS(1 + \frac{1}{n}) = \lim_{n\to\infty} A(\frac{1+\frac{1}{n}-1}{2}) = 0 = A(0)$.

$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} SA(1 + \frac{1}{n}) = \lim_{n\to\infty} S(1 + \frac{1}{n} - 1) = \frac{1}{2} = S(0)$.
But, $F(ASx_n, SAx_n, t) \neq 1$.
Thus, the pair (A, S) is weak subsequentially continuous and compatible of type (E) but not compatible. Hence all the conditions of Corollary 3.3 are satisfied for some $k \in (0, 1)$ and 1 is a unique common fixed point.

Example 3.14. Let $X = [0, \infty)$ with the usual metric d and (X, F, Δ) be the induced Non Archimedean Menger PM-space with $g(t) = 1 - t, \forall t > 0$ and $F(x, y, t) = H(t - d(x, y)), \forall x, y \in X, \forall t > 0$ and $\Delta(a, b) = \min\{a, b\}, \forall a, b \in [0, 1]$. Set $A = B$ and $S = T$. Define the self mappings A and S as follows:

$$A(X) = \begin{cases} \frac{x}{2}, & \text{if } x \in [0,\ 1]; \\ 3x - 2, & \text{if } x \in (1, \infty). \end{cases}$$

$$S(X) = \begin{cases} \frac{x}{3}, & \text{if } x \in [0,\ 1]; \\ 2x - 1, & \text{if } x \in (1, \infty). \end{cases}$$

Let us consider a sequence $\{x_n\} = \{\frac{1}{n}\}_{n \in N}$ in X. Then
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty}(\frac{1}{n}) = 0 = \lim_{n\to\infty} Sx_n$. Also,
$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} AS(\frac{1}{n}) = \lim_{n\to\infty} A(\frac{\frac{1}{n}}{3}) = 0 = A(0)$.

$\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} A(\frac{1}{n}) = 0 = S(0)$.

Therefore, (A, S) is weak subsequentially continuous and compatible of type (E). Next, consider another sequence $\{x_n\} = \{1 + \frac{1}{n}\}_{n\in N}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} 3(1 + \frac{1}{n}) - 2 = 1 = \lim_{n\to\infty} Sx_n$.

Also,

$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} AS(1 + \frac{1}{n}) = \lim_{n\to\infty} A(1 + \frac{2}{n}) = 1 \neq A(1)$.

$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} SA(1 + \frac{1}{n}) = \lim_{n\to\infty} S(1 + \frac{3}{n}) = 1 \neq S(1)$.
Thus, the pair (A, S) is weak subsequentially continuous and compatible of type (E) but not reciprocally continuous. Hence all the conditions of Corollary 3.3 are satisfied for some $k \in (0, 1)$ and 0 is a unique common fixed point.

Conflict of Interests
The author declare that there is no conflict of interests regarding the publication of this paper.

References

[1] Achari J. , Fixed Point theorems for a class of mappings on non-Archimedean probabilistic metric spaces, Mathematica, 25,(1983), 5-9.
[2] Beloul S. : Common fixed point theorems for weakly sub-sequentially continuous generalized contractions with applications, App. Math. E-notes, 15 (2015), 173-186.
[3] Bouhadjera H. : More general common fixed point theorems under a new concept, Demonst. Math., 49(1)(2016), 64-78.
[4] Bouhadjera H. and Thobie C.G. : Common fixed point for pair of sub-compatible maps , Hal-00356516. 1 (2009), 1-16.
[5] Chang, S.S. : Fixed point theorems for single-valued and multi-valued mappings in non-Archimedean Menger probabilistic metric spaces, Math. Japonica, 35 (5) (1990), 875-885.
[6] Ciric, Lj. B. : On fixed points of generalized contractions of probabilistic metric spaces, Publ. Inst. Math. Beograd, 18 (32) (1975), 71-78.
[7] Chauhan S., Pant B.D. and I.Mohammad : Coincidence and common fixed point theorems in Non-Archimedean Menger PM-spaces, Cubo A Math. J., 15(3)(2013), 31-44.
[8] Chang, S. S. Chen, Y. Q. and Guo, J. L.: Ekeland's variational principle and Caristi's fixed point theorems in probabilistic metric spaces, Acta Math. Appl. Sincica , 7 (3) (1991), 217-229.
[9] Cho, Y. J., Ha, K.S. and Chang, S. S. : Common fixed point theorems for compatible mappings of type (A) in non-Archimedean Menger PM-spaces, Math. Japonica, 46 (1) (1997), 169-179.
[10] Hadzic, O. : A note on IstratescuSs fixed point theorem in Non-Archimedean Menger spaces, Bull. Math. Soc. Sci. Math. Rep. Soc. Roum. 24(72) (1980), 277-280.
[11] Istratescu V.I. : Fixed point theorems for some classes of contraction mappings on nonArchimedean probablistic spaces, Publ. Math. (Debrecen), 25(1-2) (1978), 29-34.

[12] Istratescu V. I. : On common fixed point theorems with applications to the non-Archimedean Menger spaces, Attidella Acad. Naz. Linceri, 58 (1975), 374-379.
[13] Istratescu V.I. and Crivat N. : On some classes of non-Archimedean probabilistic metric spaces, Seminar de spatii metrice probabiliste, Univ. Timisoara, Nr. 12 (1974).
[14] Jungck, G. : Compatible mappings and common fixed points, Internat. J. Math. Math. Sci., 9(1986), 771-779.
[15] Jungck G., and Rhoades, B. E. : Fixed point for set valued functions without continuity, Ind. J. Pure Appl. Maths., 29 (3) (1998), 227-238.
[16] Khan M.A.and Sumitra : A common fixed point theorem in Non-Archimedean Menger PM-space, Novi Sad. J. Math., 39(1)(2009), 81-87.
[17] Kutukcu S., and Sharma S.: A common fixed point theorem in Non-Archimedean Menger PM-space, Demonst. Math., 42(4)(2009), 837-849.
[18] Menger K. : Statistical Metrics, Pro. Nat. Acad. Sci. USA, 28 (3), (1942), 535- 537.
[19] Mishra, S.N. : Common fixed points of compatible mappings in PM- spaces, Math. Japonica, 36(2)(1991),283-289.
[20] Pant R.P. : Common fixed point theorems under a new continuity condition, Ann. Univ. Ferrara Sez. VII Sci. Math. 58(1), (2012),127-141.
[21] Radu, V. : On some contraction principle in Menger spaces, An Univ. Timisoara, stiinte Math., 22 (1984), 83-88.
[22] Rao K.P.R., and Ramudu E.T. : Common fixed point theorem for four mappings in Non-Archimedean Menger PM-spaces, Filomat, 20(2)(2006), 107-113.
[23] Schweizer, B. and Sklar, A. : Statistical metric spaces, Pacific. J. Math., 10(1960), 313-334.
[24] Sehgal V.M. and Bharucha-Reid A.T. : Fixed points of contraction mappings on probabilistic metric spaces, Math. Systems Theory 6 (1972), 97-102.
[25] Schweizer, B. and Sklar, A. : Probabilistic metric spaces, Amsterdam; North Hollend, (1983).
[26] Singh, S.L. and Pant, B.D. : Fixed point theorems for commuting mappings in probabilistic metric space, Honam Math. J., 5 (1983),139-150.
[27] Stojakovic, M. : Fixed point theorem in probabilistic metric spaces, Kobe J. Math., 2 (1985), 1-9.
[28] Stojakovic, M. : Common fixed point theorem in complete metric and probabilistic metric spaces, Bull. Austral. Math. Soc., 36 (1987), 73-88.
[29] Stojakovic, M. : A common fixed point theorem in probabilistic metric spaces and its applications, Glasnic Math., 23 (1988), 203-211.
[30] Sessa, S. : On a weak commutativity condition of mappings in a fixed point considerations, Publ. Inst. Math., 32 (46) (1986), 149-153.
[31] Singh, M.R. and Mahendra, S. Y. : Compatible mappings of type (E) and common fixed point theorems of Meir-Keller type, Int. J. Math. Sci. Engg. Appl., 1(2) (2007), 299-315.
[32] Singh S.L. and Pant B.D. : Common fixed points of weakly commuting mappings on Non-Archimedean Menger PM- spaces, Vikram J. Math., 6 (1985/86), 27-31.
[33] Singh S.L. and Pant B.D. : Fixed points theorems in weakly commuting mappings on Non-Archimedean Menger PM- spaces, J. Nonlinear Anal.Optim. Theo. App., 6 (1985/86), 27-31.
[34] Thagafi, A. M.A and Shahzad, N. : Generalized I-nonexpansive selfmaps and invariant approximations, Acta. Math. Sin. (Engl. Ser.),24(50) (2008), 867-876.

[35] Sharma, R. : Weak sub sequential continuous mapsin non Archimedean Menger PM space, Bull. Int. Math. Virt. Inst., 7 (2017), 65-72. www.imvibl.org /JOURNALS / BULLETIN

Bangmod International
Journal of **M**athematical **C**omputational **S**cience
ISSN: 2408-154X
Bangmod-JMCS Online @ http://bangmod-jmcs.kmutt.ac.th/

Journal office:
Theoretical and Computational Science Center (TaCS)
Science Laboratory Building, Faculty of Science
King Mongkuts University of Technology Thonburi (KMUTT)
126 Pracha Uthit Road, Bang Mod, Thung Khru, Bangkok, Thailand 10140
Website: http://tacs.kmutt.ac.th/
Email: tacs@kmutt.ac.th