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SOLUTION OF INTEGRAL EQUATION INVOLVING INTERPOLATIVE ENRICHED CYCLIC KANNAN CONTRACTION MAPPINGS

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Abstract The purpose of this paper is to introduce the class of interpolative enriched cyclic Kannan contraction mappings defined on an Banach space and to prove the existence and uniqueness of fixed point of the such mappings. An example is presented to support the concept introduced herein. Moreover, an application of the main result to solve nonlinear integral equations is also given. Our result extend and generalize various results in the existing literature.

MSC: 47H09; 47H10

Keywords: Fixed point; interpolative Kannan contraction; enriched cyclic contraction; integral equation

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1. Introduction and Preliminaries

Let (X, d) be any metric space. A mapping $T : X \to X$ is called contraction mapping if there exists a constant $a \in [0, 1)$ such that for all $x, y \in X$, we have

$$d(Tx, Ty) \le ad(x, y). \tag{1.1}$$

One of the most important results used in metric fixed point theory is the well known Banach contraction principle which states that any contraction mapping on a complete metric space (X, d) has a unique fixed point. Clearly, contraction is always continuous on X. It is a matter of great interest to study contractive conditions which do not imply the continuity of T on X. Kannan [26] in 1968 proved a fixed point theorem for Kannan contraction mapping that do not need be a continuous. Recall that, a mapping $T : X \to X$ is called Kannan contraction mapping if there exists a constant $a \in [0, \frac{1}{2})$ such that for all $x, y \in X$, we have

$$d(Tx, Ty) \le a[d(x, Tx) + d(y, Ty)]. \tag{1.2}$$

Subsequently, it initiated the study of contractive type conditions that do not imply the continuity of T. For more results in this direction, we refer to [4–7, 14, 15, 24, 27, 32, 35–37] and references therein.

Recently, Karapainar in [27] extended the class of Kannan contraction mappings by introducing the class of interpolative Kannan type contraction mappings.

A mapping $T : X \to X$ is called an interpolative Kannan type contraction [27] if there exist $a \in [0, 1)$ and $\alpha \in (0, 1)$ such that for all $x, y \in X \setminus Fix(T) = \{x \in X : x = Tx\}$, we have

$$d(Tx, Ty) \le a[d(x, Tx)]^{\alpha}[d(y, Ty)]^{1-\alpha}. \tag{1.3}$$

It was proved that any interpolative Kannan type contraction mapping defined on a complete metric space has a unique fixed point [27]. For more results in this direction, see ([28, 29]).

It is worth mentioning that the mappings satisfying certain contractive conditions are self mappings on their domain of definition. Rhoades [34] obtained a fixed point result for nonself contractive type mappings, which was later modified by Ćirić [23] (see also, [10]). Some interesting fixed point results have been obtained in this direction, see for example, [8, 9, 12, 22, 25].

In 2003, Kirk et al., [30] considered a cyclic representation of the space with respect to a discontinuous mapping and extended Banach contraction principle. Let X be a nonempty set, p a positive integer, and T a self mapping on X. A finite collection $\{S_j \subseteq X : j = 1, 2, 3, \dots, p\}$ is called a cyclic representation of X with respect to T if

(1) $X = \bigcup_{j=1}^{p} S_j$;
(2) $T(S_1) \subseteq S_2, \dots, T(S_{p-1}) \subseteq S_p$, and $T(S_p) \subseteq S_1$.

The fixed point theorem in [30] is stated as follows.

Theorem 1.1. [30] *Let (X, d) be a complete metric space, p a positive integer, $\{S_1, \dots, S_p\}$ a finite family of nonempty closed subsets of X, and $T : \bigcup_{j=1}^{p} S_j \to \bigcup_{j=1}^{p} S_j$. Assume that:*

(1) *$\{S_j : j = 1, 2, 3, \dots, p\}$ is cyclic representation of $\bigcup_{j=1}^{p} S_j$ with respect to T;*

(2) *there exists* $k \in [0, 1)$ *such that for* $x \in S_j$ *and* $y \in S_{j+1}$*, we have*
$d(Tx, Ty) \leq kd(x, y)$,
where $S_{p+1} = S_1$*. Then* T *has unique fixed point* x^* *in* $\bigcap_{j=1}^{p} S_j$*.*

Let D be a convex subset of a normed space X, $\lambda \in (0, 1]$ and $T : D \to D$. A mapping $T_\lambda : D \to D$ given by

$$T_\lambda(x) = (1 - \lambda)x + \lambda Tx$$

is called an averaged mapping. Note that, the set of all fixed points of an averaged mapping coincides with set of all fixed points of T.
There arises a question that, if the collection $\{S_1, ..., S_p\}$ of nonempty closed subsets of a normed space $(X, \|\cdot\|)$ is a cyclic representation of $\bigcup_{j=1}^{p} S_j$ with respect to T_λ for some λ and $T : \bigcup_{j=1}^{p} S_j \to X$ satisfies certain contractive condition. Then, whether such an operator T possesses a fixed point or not?
This was answered in an affirmative way by Abbas et al., by proving a fixed point result for generalized enriched cyclic contractions [1].
For more discussions on enriched contractions mappings, we refer to [2, 3, 11, 13, 16–21, 31] and references therein.

Motivated by the idea of Abbas et al. [1] and Karapinar [27], we introduce the class of interpolative enriched cyclic Kannan contraction mappings and prove a fixed point result in the frame work of complete metric spaces. An example is presented to support the result proved herein. As an application of our result, we obtain the existence and uniqueness of the solution of a class of nonlinear integral equations involving interpolative enriched cyclic Kannan contraction mappings .

2. Main results

In this section, we introduce the concept of interpolative enriched cyclic Kannan contraction mappings and obtain existence and approximation results of such mappings. Throughout this section, $\{S_1, \ldots, S_p\}$ denotes a finite family of nonempty closed subsets of a normed space $(X, \|\cdot\|)$, where p is some positive integer. The symbols $\mathbb{N}$ and $\mathbb{R}$ denote the set of all natural numbers and the set of all real numbers, respectively.

Definition 2.1. A mapping $T : \bigcup_{j=1}^{p} S_j \to X$ is called an interpolative enriched cyclic Kannan contraction if it satisfies the following conditions:

(1) $\{S_j : j = 1, 2, 3, \ldots, p\}$ is cyclic representation of $\bigcup_{j=1}^{p} S_j$ with respect to T_λ.
(2) There exist $b \in [0, \infty), a \in [0, 1)$ and $\alpha \in (0, 1)$ such that for all $x \in S_j, y \in S_{j+1}$ for $1 \leq j \leq p$,

$$\|b(x - y) + Tx - Ty\| \leq a \|x - Tx\|^{\alpha} \|y - Ty\|^{1-\alpha}, \tag{2.1}$$

where $\lambda = \frac{1}{b+1}$.

To highlight the constants involved in (2.1), we call interpolative enriched cyclic Kannan contraction T, a (b, a, α)-interpolative enriched cyclic Kannan contraction.

We now present our main result.

Theorem 2.2. *If* $T : \bigcup_{j=1}^{p} S_j \to X$ *is a* (b, a, α)*-interpolative enriched cyclic Kannan contraction. Then*

(1) *$Fix(T) = \{x^*\}$, for some $x^* \in \bigcap_{j=1}^{p} S_j$;*
(2) *There exists $\lambda \in (0,1]$ such that an iterative scheme $\{x_n\}_{n=0}^{\infty}$, given by*

$$x_{n+1} = (1-\lambda)x_n + \lambda T x_n, \quad n \geq 0, \tag{2.2}$$

converges to x^* for any $x_0 \in \bigcup_{j=1}^{p} S_j$;

Proof. By the (b, a, α)-interpolative enriched cyclic Kannan contraction condition (2.1), we have

$$\left\| \left(\frac{1}{\lambda} - 1\right)(x-y) + Tx - Ty \right\| \leq a \|x - Tx\|^{\alpha} \|y - Ty\|^{1-\alpha},$$

which can be written in an equivalent form as follows:

$$\|T_\lambda x - T_\lambda y\| \leq a \|x - T_\lambda x\|^{\alpha} \|y - T_\lambda y\|^{1-\alpha}. \tag{2.3}$$

Let $x_0 \in \bigcup_{j=1}^{p} S_j$. Then there exists $j \in \{1, \ldots, p\}$ such that $x_0 \in S_j$. As $\{S_j : j = 1, 2, 3, \ldots, p\}$ is a cyclic representation of $\bigcup_{j=1}^{p} S_j$ with respect to T_λ, we have $x_1 = T_\lambda x_0 \in S_{j+1}$. From (2.3), we get

$$\|T_\lambda x_0 - T_\lambda x_1\| \leq a \|x_0 - T_\lambda x_0\|^{\alpha} \|x_1 - T_\lambda x_1\|^{1-\alpha},$$

which gives that

$$\|x_1 - x_2\| \leq a \|x_0 - x_1\|,$$

where $x_2 = T_\lambda x_1$. By induction, we obtain that

$$\|x_n - x_{n+1}\| \leq a^n \|x_0 - x_1\|.$$

Now, for any numbers $n, m \in \mathbb{N}$ with $m > 0$, we have

$$\|x_n - x_{n+m}\| \leq \sum_{k=n}^{n+m-1} \|x_k - x_{k+1}\| \leq a^n \left(\frac{1-a^m}{1-a}\right) \|x_1 - x_0\|. \tag{2.4}$$

Since $a \in [0,1)$, the sequence $\{x_n\}_{n=0}^{\infty}$ is a Cauchy sequence in $\bigcup_{j=1}^{p} S_j$. As $\bigcup_{j=1}^{p} S_j$ is complete subspace of X, $\{x_n\}_{n=0}^{\infty}$ converges to some point $x^* \in \bigcup_{j=1}^{p} S_j$. By the fact that $\{S_j : j = 1, 2, 3, \ldots, p\}$ is cyclic representation of $\bigcup_{j=1}^{p} S_j$ with respect to T_λ, the sequence $\{x_n\}_{n=0}^{\infty}$ has infinite number of terms in S_j for each $j \in \{1, ..., p\}$. Therefore $x^* \in \bigcap_{j=1}^{p} S_j$. We now prove that x^* is the fixed point of T_λ. It follows from (2.3) that

$$\begin{aligned}
\|x^* - T_\lambda x^*\| &\leq \|x^* - x_{n+1}\| + \|T_\lambda x^* - x_{n+1}\| \\
&= \|x^* - x_{n+1}\| + \|T_\lambda x^* - T_\lambda x_n\| \\
&\leq \|x^* - x_{n+1}\| + a \|x^* - T_\lambda x^*\|^{\alpha} \|x_n - T_\lambda x_n\|^{1-\alpha}.
\end{aligned}$$

This gives

$$\|x^* - T_\lambda x^*\| \leq \|x^* - x_{n+1}\| + a \|x^* - T_\lambda x^*\|^{\alpha} \|x_n - x_{n+1}\|^{1-\alpha}.$$

On taking limit as $n \to \infty$, we obtain that $\|x^* - T_\lambda x^*\| = 0$, and hence x^* is the fixed point of T_λ. To prove the uniqueness of x^*; let $p^* \in \bigcap_{j=1}^{p} S_j$ be such that $T_\lambda p^* = p^*$. By (2.3), we have

$$\begin{aligned}
\|x^* - p^*\| &= \|T_\lambda x^* - T_\lambda p^*\| \\
&\leq a \|x^* - T_\lambda x^*\|^{\alpha} \|p^* - T_\lambda p^*\|^{1-\alpha}.
\end{aligned}$$

From the above inequality we have, $\|x^* - p^*\| = 0$, that is, $x^* = p^*$. ∎

We obtain Theorem 2.0.4 of [1] as a corollary of our result .

Corollary 2.3. [1] *Let* $(X, \|\cdot\|)$ *be a Banach space and* $T : \bigcup_{j=1}^{p} S_j \to X$ *a* $(b, 0, a)$*-generalized enriched cyclic contraction. Then,* T *has a unique fixed point.*

Proof. The result follows from Theorem 2.2. ∎

If we take $b = 0$ in Theorem 2.2, we obtain the following result.

Corollary 2.4. *Let* $(X, \|\cdot\|)$ *be a Banach space and* $T : \bigcup_{j=1}^{p} S_j \to X$*. Assume that:*

1. $\{S_j : j = 1, 2, 3, \ldots, p\}$ *is cyclic representation of* $\bigcup_{j=1}^{p} S_j$ *with respect to* T*,*
2. *there exist* $a \in [0, 1)$ *and* $\alpha \in (0, 1)$ *such that for all* $x \in S_j, y \in S_{j+1}$ *for* $1 \leq j \leq p$*,*

$$\|Tx - Ty\| \leq a \|x - Tx\|^{\alpha} \|y - Ty\|^{1-\alpha}. \tag{2.5}$$

If we take $\bigcup_{j=1}^{p} S_j = X$ in Corollary 2.4, we obtain Theorem 2.2 of [27] in the setting of Banach space.

Corollary 2.5. *Let* $(X, \|\cdot\|)$ *be a Banach space and* $T : X \to X$ *satisfies*

$$\|Tx - Ty\| \leq a \|x - Tx\|^{\alpha} \|y - Ty\|^{1-\alpha},$$

for all $x, y \in X$ *such that* $Tx \neq x$ *whenever* $Ty \neq y$*, with* $a \in [0, 1)$ *and* $\alpha \in (0, 1)$*. Then* T *has a unique fixed point.*

We now present an example to illustrate Theorem 2.2.

Example 2.6. Let $X = \mathbb{R}^2$ be endowed with the usual norm. Define $T : S_1 \cup S_2 \to \mathbb{R}^2$ by

$$Tx = \begin{cases} (-x_1, x_2) & \text{if } x = (x_1, x_2) \in S_1 \\ (x_1, -x_2) & \text{if } x = (x_1, x_2) \in S_2, \end{cases}$$

where

$$S_1 = \{(x, 0); \quad x \in \mathbb{R}\} \text{ and } S_2 = \{(0, x); \quad x \in \mathbb{R}\}.$$

If $b = 1$, then $\lambda = \frac{1}{2}$ and we have

$$T_{\frac{1}{2}}x = \begin{cases} (0, x_2) & \text{if } x = (x_1, x_2) \in S_1, \\ (x_1, 0) & \text{if } x = (x_1, x_2) \in S_2. \end{cases}$$

It is easy to check that $\{S_1, S_2\}$ is cyclic representation of $S_1 \cup S_2$ with respect $T_{\frac{1}{2}}$. Note that, T is $\left(1, \frac{1}{3}, \frac{1}{2}\right)$-interpolative enriched cyclic Kanna contraction. Following arguments similar to those given in the proof of Theorem 2.2, we observe that (2.1) is equivalent to (2.3) which becomes

$$\left\|T_{\frac{1}{2}}x - T_{\frac{1}{2}}y\right\| \leq \frac{1}{3} \left\|x - T_{\frac{1}{2}}x\right\|^{\frac{1}{2}} \left\|y - T_{\frac{1}{2}}y\right\|^{\frac{1}{2}}.$$

Indeed, for any $x \in S_1$ and $y \in S_2$, we have

$$\begin{aligned} \|(0,0) - (0,0)\| &\leq \frac{1}{3}\Big(\|(x_1, 0)\|^{\frac{1}{2}} (\|(0, y_2)\|^{\frac{1}{2}} \Big) \\ 0 &\leq \frac{1}{3}|x_1|^{\frac{1}{2}}|y_2|^{\frac{1}{2}}. \end{aligned}$$

So T satisfies all the conditions of Theorem 2.2. Moreover, $x^* = (0,0) \in S_1 \cap S_2$ is the fixed point of T.

3. Application

In this section, we apply Theorem 2.2 to study the existence and uniqueness of solution of nonlinear integral equations. We consider the nonlinear integral equation given by

$$u(t) = \int_0^{\vartheta} G(t,s) f(s,u(s))ds; \quad t \in [0,\vartheta], \tag{3.1}$$

where $\vartheta > 0$, $f : [0,\vartheta] \times \mathbb{R} \to \mathbb{R}$ and $G : [0,\vartheta] \times [0,\vartheta] \to [0,\infty)$ are continuous functions. Let $X = C([0,\vartheta])$ be a set of real continuous functions defined on $[0,\vartheta]$ and $d : X \times X \to \mathbb{R}$ be defined by

$$d_\infty(f,g) = \max_{t\in[0,\vartheta]} |f(t) - g(t)|, \quad f,g \in X. \tag{3.2}$$

It is known that (X, d_∞) is a complete metric space. Let $g,h \in X$ and $\alpha_0, \beta_0 \in \mathbb{R}$ be such that

$$\alpha_0 \le g(t) \le h(t) \le \beta_0, \quad t \in [0,\vartheta]. \tag{3.3}$$

Suppose that for all $t \in [0,\vartheta]$, $u \in C([0,\vartheta])$ and $\lambda \in (0,1]$, we have

$$g(t) \le \lambda \int_0^{\vartheta} G(t,s) f(s,h(s))ds + (1-\lambda)u(s), \tag{3.4}$$

and

$$h(t) \ge \lambda \int_0^{\vartheta} G(t,s) f(s,g(s))ds + (1-\lambda)u(s). \tag{3.5}$$

Moreover, for each $s \in [0,\vartheta]$,the mapping $f(s,\cdot)$ is a non-increasing function, that is,

$$\text{for all } x,y \in \mathbb{R}, \text{ with } x \ge y, \text{ we have } f(s,x) \le f(s,y), \text{ for each } s \in [0,\vartheta]. \tag{3.6}$$

Also,

$$\sup_{t\in[0,\vartheta]} \int_0^{\vartheta} G(t,s)ds \le 1. \tag{3.7}$$

Finally, suppose that, for $\lambda \in (0,1], a \in [0,1), \alpha \in (0,1)$, and for all $s \in [0,\vartheta]$, $x,y \in \mathbb{R}$ with $(x \le \beta_0 \ \text{ and } \ y \ge \alpha_0)$ or $(x \ge \alpha_0 \ \text{ and } \ y \le \beta_0)$, we have

$$|f(s,x) - f(s,y)| \le \frac{a}{\lambda}\Big(|x - Tx|^{\alpha}|y - Ty|^{1-\alpha} - (1-\lambda)(x-y)\Big). \tag{3.8}$$

Let us consider the set

$$\mathsf{C} =: \{u \in C([0,\vartheta]) : \quad g(t) \le u(t) \le h(t); \quad t \in [0,\vartheta]\}. \tag{3.9}$$

We have the following result.

Theorem 3.1. *Under the assumption (3.3-3.9), the nonlinear integral equation (3.1) has a unique solution $u^* \in \mathsf{C}$.*

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Proof. Define A_h and A_g by

$$A_h = \{u \in X : u(s) \leq h(s), \quad \forall s \in [0, \vartheta]\}$$
$$A_g = \{u \in X : u(s) \geq g(s), \quad \forall s \in [0, \vartheta]\}$$

which are the closed subsets of X. Define the mapping $T : A_h \cup A_g \to X$ by

$$Tu(t) = \int_0^\vartheta G(t,s)f(s,u(s))ds; \quad t \in [0, \vartheta].$$

For $\lambda \in (0, 1]$, we have

$$T_\lambda u(t) = \lambda \int_0^\vartheta G(t,s)f(s,u(s))ds + (1-\lambda)u(t); \quad t \in [0, \vartheta].$$

Let $u \in A_h$. By using (3.6), we have

$$G(t,s)f(t,u(s)) \geq G(t,s)f(t,h(s)), \quad \forall t,s \in [0,\vartheta].$$

It follows from (3.4) that

$$\begin{aligned}\lambda \int_0^\vartheta G(t,s)f(t,u(s))ds + (1-\lambda)u(t) &\geq \lambda \int_0^\vartheta G(t,s)f(t,h(s))ds + (1-\lambda)u(t)\\ &\geq g(t).\end{aligned}$$

Thus, $T_\lambda(u) \geq g$ and we obtain that

$$T_\lambda(A_h) \subseteq A_g.$$

Similarly, we have

$$T_\lambda(A_g) \subseteq A_h.$$

By conditions (3.7) and (3.8), we have

$$\begin{aligned}|T_\lambda u - T_\lambda v| &\leq \lambda \int_0^\vartheta G(t,s)\Big(|f(s,u(s)) - f(t,v(s))|\\ &\qquad + (1-\lambda)(u(s)-v(s))\Big)ds\\ &\leq a\int_0^\vartheta G(t,s)|u-Tu|^\alpha |v-Tv|^{1-\alpha}ds,\end{aligned}$$

$$\begin{aligned}\max_{t\in[0,\vartheta]} |T_\lambda u(t) - T_\lambda v(t)| &\leq a\Big(\max_{t\in[0,\vartheta]} |u(t)-Tu(t)|\Big)^\alpha\\ &\qquad \Big(\max_{t\in[0,\vartheta]} |v(t)-Tv(t)|\Big)^{1-\alpha} \int_0^\vartheta G(t,s)ds.\end{aligned}$$

Therefore,

$$d_\infty(T_\lambda u, T_\lambda v) \leq a[d_\infty(u, T_\lambda u)]^\alpha [d_\infty(v, T_\lambda v)]^{1-\alpha}.$$

Since all the conditions of Theorem 2.2 are satisfied, an integral equation (3.1) has a unique solution $u^* \in \mathsf{C}$. ∎

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